The Universal Local de Sitter Anomaly Channel Is Uniquely Type A

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August 2026

Abstract

We prove that the universal local logarithmic anomaly response of a four-dimensional quantum field theory on exact round de Sitter has a one-dimensional QFT image. Within the standard parity-even Weyl-anomaly sector, the only surviving microscopic coordinate is the type-A anomaly coefficient a. In four dimensions the local anomaly can also contain the type-B Weyl term, boundary charges and scheme-dependent total derivatives. We ask which of these data can be represented both on the closed round S^4 saddle and in the $n = 1$ round-horizon response, while remaining invariant under finite counterterms and independent of the auxiliary regulating boundary. The answer is unique: the image is generated by the Euler/Wess–Zumino class. Bulk type-B data, boundary anomaly charges, particle masses and couplings do not enter as independent coordinates of this projected sector.

The static-patch representative provides a non-trivial boundary-sector test. The non-Euler invariant j_2 is present and nonzero on the auxiliary worldtube. Its integrated geometrical coefficient depends singularly on the tube geometry as the horizon is approached, while its anomaly coefficient can depend on the imposed boundary condition. It is therefore genuine data of the specified smooth-boundary problem, but has no independent image in the original no-boundary round-horizon logarithm. The horizon leg is a stress test of the compact selection, not an independent derivation of the standard spherical $4a$ coefficient.

At a conformal fixed point the classification applies directly; away from one, beta-function and operator-mixing responses must first be separated by the local-RG projection. At one loop the result follows from the heat-kernel and local-counterterm classification; beyond one loop its structural content follows from Weyl cohomology and Wess–Zumino consistency. For the unbroken minimal Standard Model free-field census,

$$a_{\rm SM} = \frac{1991}{720} \simeq 2.77.$$

Any observable whose microscopic dependence genuinely factors solely through this channel can depend on the QFT only through a. The theorem identifies the protected matter coordinate available to de Sitter anomaly-channel constructions; it does not determine the gravity-side scale-setting rule or the value of Λ.

Keywords: conformal anomaly; de Sitter space; cosmological constant; Euclidean gravity; entanglement entropy; Standard Model; type-A anomaly

1 Introduction

This paper proves that exact round de Sitter has a one-dimensional QFT image in its universal local logarithmic anomaly channel. The full renormalised QFT effective action remains a many-parameter object and can depend on masses, couplings, non-local terms, stress-tensor data,

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boundary observables and the full anomaly multiplet. The question here is narrower: in four dimensions, which local anomaly data are common to the closed round sphere and the $n = 1$ round horizon while remaining invariant under finite counterterms, trivial Weyl cocycles and auxiliary regulator-boundary choices? At a conformal fixed point this is the standard anomaly classification; away from one, beta-function and operator-mixing responses are separated before the projection is applied. We call the resulting object the universal local de Sitter scale-response sector.

The answer is exactly one coordinate. We denote the microscopic QFT by Q and its type-A anomaly coefficient by $a(Q)$. Let ρ_{S^4} and $\rho_{\rm hor}$ evaluate a common local anomaly class on the closed round sphere and the $n = 1$ round horizon, and let $\rho_{\rm dS} = \rho_{S^4} \oplus \rho_{\rm hor}$. The cohomological projection $\Pi^{\rm univ}_{\rm dS}$, defined in Section 2, has

$$\mathrm{Im}\left(\Pi^{\rm univ}_{\rm dS} \circ \rho_{\rm dS}\Big|_{\mathcal{V}_{\rm loc}} \right) = \mathrm{span}\{\mathbf{e}^{\rm dS}_A\}, \tag{1}$$

where $\mathbf{e}^{\rm dS}_A$ is the unit-normalised generator of the quotient line represented by the Euler density and its Chern–Gauss–Bonnet/ Wess–Zumino completion. With the integrated-anomaly convention in Eq. (7), the corresponding scalar coordinate is

$$\mathfrak{a}_{\rm dS}(Q) = a(Q). \tag{2}$$

Thus the universal local logarithmic response of exact round de Sitter transmits the type-A anomaly coefficient and no second independent QFT coordinate.

The ingredients entering this statement are individually standard: the four-dimensional anomaly basis, the finite-counterterm classification, the spherical $4a$ logarithm and the allowed boundary anomaly charges. The new result is their cross-realisation intersection. The channel is defined without presupposing which cohomologically non-trivial anomaly class survives. Each competing sector is then eliminated for a distinct reason: C^2 and j_1 vanish by conformal flatness; the total-derivative sector is a counterterm coboundary; and the nonzero j_2 sector belongs to the specified auxiliary codimension-one boundary problem rather than to the common no-boundary response. The Euler/Wess–Zumino class is the unique surviving image.

The two de Sitter representatives play complementary roles. The closed round saddle $S^4(H)$ supplies the compact constant-Weyl extraction and its counterterm uniqueness [7]. The round static-patch horizon supplies the observer-centred representative of the same anomaly class [28, 30, 31, 45]. The horizon leg is not used to rederive the standard spherical coefficient $4a$. It tests the boundary sector in a representation where the non-Euler invariant j_2 is genuinely present on the regulated worldtube. Its integrated geometrical coefficient depends singularly on the cutoff surface, while the associated anomaly charge can depend on the boundary condition. The term is therefore real data of that smooth boundary problem, but not a second coordinate of the original no-boundary round-horizon logarithm. The use of compact S^4 is not a claim about the literal topology of the Lorentzian universe; it is the standard smooth Euclidean representative of de Sitter geometry and the compact saddle in the Gibbons–Hawking treatment.

The theorem has an immediate model-building consequence. Any observable whose microscopic dependence genuinely factors only through this channel must take the form $\mathcal{O}_{\rm dS}[Q, g] = \mathcal{F}[g; a(Q)]$; for a linear projected response this reduces to $F[g]a(Q)$. Independent dependence on c, a boundary charge, a particle mass or a Yukawa coupling therefore shows that a larger effective-action sector is being used. For the unbroken minimal Standard Model free-field census, the selected coordinate is

$$a_{\rm SM} = \frac{1991}{720} \simeq 2.77. \tag{3}$$

This is an evaluation of the theorem, not a fit to cosmology.

The cosmological-constant problem is one application, not the definition of the result. The usual zero-point presentation assigns $\frac{1}{2}\hbar\omega$ to every field mode and compares a cutoff sum with the observed dark-energy scale [1, 2]. On a single compact Euclidean saddle, however,

$$\int_M \sqrt{g}\,\rho_{\rm vac} \qquad \text{and} \qquad \alpha \int_M \sqrt{g} \tag{4}$$

are the same local metric functional; only their renormalised sum enters the gravitational equations. A compact de Sitter construction that reads matter solely through the channel classified here therefore uses the type-A coefficient a, rather than a separately defined absolute zero-point density, as its protected matter input. The conversion of this dimensionless coordinate into a physical cosmological scale, and its radiative stability, remain gravity-side scale-setting and dynamics problems.

Relation to recent literature

Modern work makes clear that the complete static-patch quantum problem is much richer than the single local coordinate isolated here. Algebraic and modular analyses treat the observer patch as an intrinsic quantum system, while sphere/static-patch factorisations exhibit genuine bulk and horizon-edge sectors [45, 30, 31, 46, 20, 51, 53, 55, 56, 47, 58, 57]. The present theorem does not remove or reconstruct those sectors. It classifies only the regulator-independent local logarithmic anomaly coordinate common to the closed sphere and round horizon after the full auxiliary boundary problem is set aside.

Recent de Sitter entropy calculations likewise continue to connect horizon logarithms and anomaly data [34, 35, 50, 36], while modern boundary/defect and effective-action analyses emphasise the cohomological distinction used here between genuine anomaly classes, finite-counterterm coboundaries and Weyl-invariant or non-local terms [52, 54, 25]. The contribution of this paper is to apply that bookkeeping across the two exact round de Sitter representatives and to show, including an explicit nonzero boundary-sector test, that their common local QFT image is the Euler/Wess–Zumino class alone.

Section 2 defines the cohomological de Sitter projection and proves the type-A selection theorem. Section 3 proves the compact-S^4 extraction theorem and its counterterm uniqueness. Section 4 gives the static-patch horizon realisation and the boundary-sector test. Section 5 evaluates the selected coordinate for the Standard Model. Sections 6 and 7 state the cosmological implication and the precise scope of the result.

2 The universal local de Sitter scale-response sector

2.1 The local anomaly data

For a four-dimensional conformal field theory on a smooth manifold without boundary, the local trace anomaly has the Deser–Schwimmer form [8, 9]

$$\langle T^{\mu}{}_{\mu}\rangle = \frac{1}{16\pi^2}\left(c\, C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} - a\, E_4 + b\,\Box R\right), \tag{5}$$

where

$$E_4 = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2 \tag{6}$$

is the Euler density. The coefficients a and c multiply cohomologically non-trivial anomaly classes, constrained by Wess–Zumino consistency [26]. The coefficient b is scheme-dependent because it can be shifted by a finite R^2 counterterm. The sign convention in Eq. (5) is the one for which a constant Weyl variation of the effective action on round S^4 gives $-4a\sigma$.

On a manifold with boundary, the Euler term acquires its Chern–Gauss–Bonnet boundary completion and additional boundary anomaly charges are allowed. Boundary CFT structure and the local integrated anomaly basis are standard [27, 10, 11]. In the integrated notation of Fursaev, the local logarithmic anomaly can be organised as

$$\mathcal{A}_4[M, \partial M] = 2a\,\chi(M) + c\,I[M] + q_1 j_1[\partial M] + q_2 j_2[\partial M] + \mathcal{D}[M, \partial M]. \tag{7}$$

Here I is the integrated Weyl-squared invariant, j_1 is the Weyl–extrinsic boundary invariant, j_2 is proportional to $\int_{\partial M} \sqrt{h}\,\mathrm{Tr}\,\widehat{K}^3$, and $\mathcal{D}$ denotes the scheme-dependent total-derivative sector and its boundary completion. The normalisation is chosen so that the Euler contribution on closed S^4 is $2a\chi(S^4) = 4a$.

The local anomaly space relevant to the present comparison is therefore

$$\mathcal{V}_{\mathrm{loc}} = \mathrm{span}\{\mathcal{E}_4,\, C^2,\, j_1,\, j_2,\, \mathcal{D}\}, \tag{8}$$

where $\mathcal{E}_4$ denotes the bulk Euler density together with the boundary completion required by topology and Wess–Zumino consistency. Equation (8) is not the full effective action. It omits non-local terms, massive threshold functions, higher-derivative EFT operators, defect operator data and finite boundary observables. It is the standard local four-derivative logarithmic anomaly sector.

2.2 Cohomological definition of the channel

We now define the channel without presupposing which non-trivial anomaly class survives. Let $\mathcal{P}^{(4)}_{\mathrm{loc}}$ denote the projection of a full scale response onto its parity-even local four-derivative logarithmic anomaly component. This step separates the anomaly sector from non-local terms, massive threshold functions, running-coupling responses and higher-derivative EFT data; it does not discard any class within the local anomaly basis (8). For a QFT Q, write its abstract local anomaly response as

$$\mathcal{A}^{\mathrm{loc}}_Q \equiv \mathcal{P}^{(4)}_{\mathrm{loc}}(\delta_\sigma W_Q)\Big|_{\sigma=1} \in \mathcal{V}_{\mathrm{loc}}, \tag{9}$$

where δ_σ is a constant Weyl variation.

The same local anomaly class can be evaluated in the two exact round de Sitter representatives. Define linear evaluation maps

$$\rho_{S^4} : \mathcal{V}_{\mathrm{loc}} \longrightarrow \mathcal{W}_{S^4}, \qquad \rho_{\mathrm{hor}} : \mathcal{V}_{\mathrm{loc}} \longrightarrow \mathcal{W}_{\mathrm{hor}}, \tag{10}$$

where ρ_{S^4} gives the closed-S^4 constant-Weyl response and ρ_{hor} gives the universal logarithmic spherical-horizon entropy response evaluated at $n = 1$. The paired evaluation and its image are

$$\rho_{\mathrm{dS}} \equiv \rho_{S^4} \oplus \rho_{\mathrm{hor}}, \qquad \mathcal{W}_{\mathrm{dS}} \equiv \mathrm{Im}\,\rho_{\mathrm{dS}} \subset \mathcal{W}_{S^4} \oplus \mathcal{W}_{\mathrm{hor}}. \tag{11}$$

Thus $\mathcal{W}_{\mathrm{dS}}$ is the space of paired local responses induced by a common anomaly class. This definition does not identify the complete sphere and static-patch partition functions.

Two classes of terms carry no universal de Sitter information. First are the Weyl variations of finite local counterterms. These are moved by a choice of renormalisation scheme. Second are cohomologically trivial local Weyl cocycles, i.e. coboundaries of the Weyl differential, whose triviality is fixed by Wess–Zumino consistency. In the horizon representative there is a third auxiliary class: local tube data that fail one or more of the common-channel requirements. Concretely, these are terms whose value changes under symmetry-preserving changes of the regulator-tube location or regulator boundary condition, which have no representative on the closed saddle, or which have no independent meaning in the original no-boundary $n = 1$ spherical-horizon observable.

Definition 1 (Cohomological de Sitter projection)**.** *Let $\mathcal{W}_{\mathrm{triv}} \subset \mathcal{W}_{\mathrm{dS}}$ be the linear span of paired responses that are cohomologically trivial, including finite local counterterm variations.*

Let $\mathcal{W}_{\rm aux} \subset \mathcal{W}_{\rm dS}$ be the linear span of paired responses whose horizon-tube component fails one or more of the common-channel requirements above. The universal local de Sitter projection is the quotient map

$$\Pi_{\rm dS}^{\rm univ} : \mathcal{W}_{\rm dS} \longrightarrow \mathcal{W}_{\rm dS}/(\mathcal{W}_{\rm triv} + \mathcal{W}_{\rm aux}). \tag{12}$$

For a QFT Q, the associated projected response is the quotient class

$$[\mathcal{R}_Q]_{\rm dS} \equiv \Pi_{\rm dS}^{\rm univ}\, \rho_{\rm dS}(\mathcal{A}_Q^{\rm loc}). \tag{13}$$

The two components of $\rho_{\rm dS}$ implement the closed-sphere and round-horizon scale variations, respectively.

Definition 1 prespecifies the parity-even local four-derivative logarithmic anomaly sector, but it does not select the Euler density or any other non-trivial class. It first divides out precisely those local responses that can be changed by counterterms, trivial Weyl cocycles, or auxiliary boundary choices, and then asks what local class remains. The Euler statement is recovered only after the theorem is proved: the surviving class is the Euler/Wess–Zumino class, whose representatives are the undressed anomaly logarithm on S^4 and the round-horizon logarithm in the static patch.

The word universal is used in this precise sense. The coefficient is independent of finite local counterterms, regulator conventions and auxiliary boundary data, and it is represented both by the closed-sphere constant-Weyl response and by the round-horizon logarithm. This is a uniqueness statement inside the local logarithmic de Sitter scale-response sector. If the closed-saddle condition is dropped, boundary-only anomaly charges can survive. If regulator independence is dropped, the full worldtube effective action can depend on q_2, on the cutoff radius and on a chosen boundary condition. Those are legitimate observables of a specified boundary problem; they are not coordinates of the common no-boundary de Sitter scale response defined above.

The construction proceeds in two distinct stages. The local projection $\mathcal{P}_{\rm loc}^{(4)}$ first excludes non-local terms, massive-threshold responses, running-coupling contributions and higher-derivative EFT data from the domain being classified. The quotient $\Pi_{\rm dS}^{\rm univ}$ then removes finite- counterterm coboundaries, cohomologically trivial local cocycles and auxiliary regulator-boundary responses. It is therefore a cohomological classification rather than a dynamical truncation of the static-patch theory [52, 54, 25].

2.3 Type-A selection

Let $\mathcal{A}_{A=1}^{\rm loc} \in \mathcal{V}_{\rm loc}$ denote the abstract local anomaly response with unit type-A coefficient and all other anomaly coefficients set to zero. Introduce the paired Euler/Wess–Zumino generator

$$\mathbf{e}_A^{\rm dS} \equiv \frac{1}{4}\, \Pi_{\rm dS}^{\rm univ}\, \rho_{\rm dS}\Big(\mathcal{A}_{A=1}^{\rm loc}\Big)\,. \tag{14}$$

The factor 1/4 fixes a unit basis vector in the integrated-anomaly normalisation $2a\chi(S^4) = 4a$. The theorem below proves that this candidate is non-zero and spans the full quotient image.

Theorem 2 (Type-A selection)**.** *For the parity-even local logarithmic anomaly sector of a four-dimensional CFT, and more generally for the fixed-point anomaly sector of a QFT whose local Weyl data are described by the standard four-derivative basis* (8)*, the universal isotropic de Sitter projection on exact round de Sitter geometry has one-dimensional QFT image:*

$$\mathrm{Im}\bigg(\Pi_{\rm dS}^{\rm univ} \circ \rho_{\rm dS}\Big|_{\mathcal{V}_{\rm loc}}\bigg) = \mathrm{span}\{\mathbf{e}_A^{\rm dS}\}. \tag{15}$$

For a QFT Q, the projected local response is

$$[\mathcal{R}_Q]_{\rm dS} = 4a(Q)\mathbf{e}_A^{\rm dS}, \tag{16}$$

with the integrated-anomaly normalisation of Eq. (7)*. Thus the only microscopic QFT coordinate transmitted by this sector is a.*

Proof. It is enough to evaluate the paired map $\Pi^{\rm univ}_{\rm dS} \circ \rho_{\rm dS}$ on the local anomaly basis (8). The evaluation maps first produce the closed-sphere and horizon responses of each class; the quotient then acts sector by sector. We apply the quotient by finite counterterm variations, trivial Weyl cocycles and auxiliary boundary data to each local anomaly sector.

Euler sector. The round sphere has $\chi(S^4) = 2$, so the integrated Euler anomaly is non-zero on the closed saddle. The regulated horizon tube has topology $D^2 \times S^2$ and therefore

$$\chi(D^2 \times S^2) = \chi(D^2)\chi(S^2) = 2. \tag{17}$$

On a manifold with boundary the bulk Euler integral and its Chern–Gauss–Bonnet boundary term form the topological completion whose coefficient is fixed by a alone [12, 13]. The Euler class is represented in both realisations, does not acquire an independent boundary charge, and is not removable by a finite local counterterm. It survives the common-channel requirements.

Bulk type-B sector. Both the round sphere and the Euclidean static patch are conformally flat. Hence $C_{\mu\nu\rho\sigma} = 0$ and the cC^2 term vanishes in both realisations. The coefficient c is not transmitted by this channel.

First boundary-charge sector. The invariant j_1 is absent on closed S^4 and is constructed from the pullback of the ambient Weyl tensor on a boundary. It therefore vanishes on the conformally flat static-patch geometry as well.

Second boundary-charge sector. The invariant j_2 can be non-zero on the regulated worldtube. This is the non-trivial test case in the theorem: the sector is genuinely available in the regulated boundary problem, so its removal is not a consequence of vanishing or notation. It is genuine boundary anomaly data for a QFT with a specified physical boundary. The de Sitter horizon logarithm used here, however, is the no-boundary $n = 1$ spherical horizon observable regulated by an auxiliary tube. The j_2 term has no representative on closed S^4, and on the static-patch tube its value depends on the extrinsic geometry, location and boundary condition of the cutoff surface. It therefore fails both the closed-saddle and regulator-independence requirements. Its exclusion is not a statement that $q_2 j_2$ is unphysical; it is a statement that it belongs to the full regulated boundary problem rather than to the common scale-response sector.

Trivial sector. The total-derivative anomaly and its boundary completion can be shifted by finite local counterterms. They fail counterterm invariance.

The standard local anomaly basis is exhausted by these sectors. Only the Euler/Wess–Zumino class remains. With the normalisation in Eq. (7), the constant-Weyl response carries coefficient $4a$, proving Eqs. (15) and (16). □

The theorem makes the scalar extraction unambiguous. Define the coordinate functional on the one-dimensional quotient by

$$\mathrm{Coeff}_A\left(\alpha \mathbf{e}^{\rm dS}_A\right) = \alpha, \tag{18}$$

and the normalised de Sitter scale-response datum by

$$\mathfrak{a}_{\rm dS}(Q) \equiv \frac{1}{4}\,\mathrm{Coeff}_A([\mathcal{R}_Q]_{\rm dS})\,. \tag{19}$$

Theorem 2 then gives

$$\mathfrak{a}_{\rm dS}(Q) = a(Q). \tag{20}$$

The action of the projection is summarised in Table 1.

Corollary 3 (One-dimensional microscopic image)**.** *Let $\mathcal{O}_{\rm dS}$ be an observable whose microscopic dependence factors entirely through the cohomological datum $\mathfrak{a}_{\rm dS}$, equivalently through the paired projection $\Pi^{\rm univ}_{\rm dS} \circ \rho_{\rm dS}$ on the local anomaly basis. Then there exists a functional $\mathcal{F}$ such that*

$$\mathcal{O}_{\rm dS}[Q, g] = \mathcal{F}[g; a(Q)]. \tag{21}$$

Sector	Closed S^4	Regulated horizon tube	Fate in the common quotient
$\mathcal{E}_4$	Present; $\chi(S^4) = 2$	Present through the Euler boundary completion; $\chi(D^2 \times S^2) = 2$	Retained; coefficient a
C^2	Zero by conformal flatness	Zero by conformal flatness	Removed
j_1	No boundary	Zero with the ambient Weyl tensor	Removed
j_2	No boundary	Generally non-zero and tied to extrinsic cutoff data	Removed from the common regulator-independent channel
$\mathcal{D}$	Zero or scheme-dependent	Scheme-dependent local sector	Removed by counterterm invariance

Table 1: Sector-by-sector action of the universal local de Sitter projection. The j_2 sector is retained in the full boundary problem but not in the common channel.

If $\mathcal{O}_{\mathrm{dS}}$ is additionally a linear response functional of the projected anomaly action, then there exists a geometric functional F such that

$$\mathcal{O}^{\mathrm{lin}}_{\mathrm{dS}}[Q, g] = F[g]\, a(Q), \tag{22}$$

up to convention-dependent normalisation. In either case the observable cannot depend independently on c, q_1, q_2, particle masses or couplings without drawing on sectors outside the channel.

This corollary is a selection rule, not a claim that every de Sitter observable factors through the channel. The full matter determinant contains much more information. The theorem says that whenever a construction is explicitly restricted to the common local logarithmic scale-response sector, the microscopic image is one-dimensional.

3 Closed S^4: the compact-saddle realisation

3.1 The single-saddle ambiguity of absolute vacuum energy

Let $\Gamma[g]$ be the renormalised matter effective action on a fixed four-dimensional background. It is defined up to local gravitational counterterms [3, 4, 5, 29],

$$\Gamma[g] \longrightarrow \Gamma[g] + \int_M \sqrt{g}\, \Big(\alpha + \beta R + \gamma R^2 + \eta R_{\mu\nu}R^{\mu\nu} + \zeta R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \xi \Box R\Big)\,. \tag{23}$$

The coefficients include divergent subtractions and arbitrary finite renormalisations. On the round sphere $S^4(H)$,

$$\mathrm{Vol}(S^4) = \frac{8\pi^2}{3H^4}, \qquad R = 12H^2, \tag{24}$$

so the cosmological term scales as H^{-4} and the Einstein–Hilbert term as H^{-2}. Curvature-squared integrals are independent of H on a round four-sphere.

An absolute zero-point contribution of the form $\rho_{\mathrm{vac}} \int \sqrt{g}$ therefore occupies exactly the same local operator as the coefficient α in Eq. (23). No operation intrinsic to one compact saddle identifies a preferred split

$$\rho_\Lambda = \rho_{\mathrm{vac}} + \alpha. \tag{25}$$

This is the compact-saddle form of the usual renormalisation ambiguity [6]. The physical quantity is the total renormalised cosmological coupling, not either term on the right-hand side of Eq. (25).

The statement is compatible with measurable Casimir forces. A Casimir observable compares configurations, and the common bulk volume term cancels from the difference [14, 15]. The compact S^4 determinant considered here is a single-saddle quantity and has no intrinsic second configuration. The absence of a separate absolute zero-point datum creates the constructive problem addressed below: what part of the matter response is invariant under the local ambiguity?

3.2 The constant-Weyl response

A constant Weyl rescaling of the round metric,

$$g_{\mu\nu} \longrightarrow \mathrm{e}^{2\sigma} g_{\mu\nu}, \tag{26}$$

acts on the curvature scale as $H \to H\mathrm{e}^{-\sigma}$. Hence

$$\delta_\sigma \Gamma[S^4(H)] = -\sigma \frac{\mathrm{d}\Gamma}{\mathrm{d}\ln H}. \tag{27}$$

On conformally flat S^4, the type-B term vanishes, R is constant, and the anomalous part of the same variation is

$$\delta_\sigma \Gamma\big|_{\mathrm{anom}} = -\frac{a}{16\pi^2}\sigma \int_{S^4} \sqrt{g}\, E_4 \tag{28}$$

$$= -\frac{a}{16\pi^2}\sigma\,(64\pi^2) = -4a\sigma, \tag{29}$$

where

$$\int_{S^4} \sqrt{g}\, E_4 = 32\pi^2 \chi(S^4) = 64\pi^2. \tag{30}$$

Comparison with Eq. (27) gives

$$\left.\frac{\mathrm{d}\Gamma}{\mathrm{d}\ln H}\right|_{\mathrm{WZ}} = 4a. \tag{31}$$

Equivalently, the anomaly-induced part of the sphere effective action contains

$$\Gamma_{\mathrm{WZ}}[S^4(H)] = -4a \ln\left(\frac{\mu}{H}\right) + \text{constant}, \tag{32}$$

up to the overall convention for $\Gamma = -\ln Z$; this is the round-sphere reduction of the anomaly-induced Wess–Zumino effective action [37, 38].

To state the extraction without circularity, let $\mathcal{W}_{S^4}$ be the space of parity-even local four-derivative logarithmic anomaly responses on the round sphere and let $\mathcal{W}_{\mathrm{triv}}$ be the subspace generated by finite local counterterm variations and cohomologically trivial local Weyl cocycles. Define the compact cohomological projection

$$\Pi_{\mathrm{coh}} : \mathcal{W}_{S^4} \longrightarrow \mathcal{W}_{S^4}/\mathcal{W}_{\mathrm{triv}} \tag{33}$$

and the projected compact response class

$$\mathcal{R}_{S^4}[\Gamma] \equiv \Pi_{\mathrm{coh}}\, \mathcal{P}^{(4)}_{\mathrm{loc}} \left(\frac{\mathrm{d}\Gamma[S^4(H)]}{\mathrm{d}\ln H} \right). \tag{34}$$

Define the unit-normalised compact Euler/Wess–Zumino generator by

$$\mathbf{e}_A^{S^4} \equiv \frac{1}{4}\, \Pi_{\mathrm{coh}}\, \rho_{S^4}\left(\mathcal{A}^{\mathrm{loc}}_{A=1}\right). \tag{35}$$

The compact projection itself is defined without selecting a surviving anomaly class. Equation (35) fixes the normalisation of the Euler/Wess–Zumino candidate. Theorem 4 proves that this candidate is the unique non-zero quotient class and identifies its coefficient with that of the undressed logarithm $\ln(\mu/H)$.

3.3 The compact extraction theorem

Theorem 4 (Compact-S^4 extraction)**.** *Let $\Gamma[g]$ be the renormalised matter effective action on round $S^4(H)$, defined up to local gravitational counterterms, and restrict its constant-Weyl response with $\mathcal{P}^{(4)}_{\text{loc}}$ to the parity-even local four-derivative logarithmic anomaly sector. For the projected class* (34)*,*

$$\mathcal{R}_{S^4}[\Gamma] = 4a\mathbf{e}_A^{S^4}. \tag{36}$$

Moreover, the Euler/Wess–Zumino class is the unique non-zero local counterterm-invariant QFT class obtained by this projected constant-Weyl response.

Proof. Existence follows directly from Eqs. (29)– (31): the trace anomaly and the Gauss–Bonnet theorem give a non-trivial Euler/Wess–Zumino constant-Weyl response with coefficient $4a$, yielding Eq. (36).

For uniqueness, consider the local counterterms in Eq. (23). On round $S^4(H)$ the cosmological term is proportional to H^{-4} and its logarithmic derivative is again proportional to H^{-4}. The Einstein–Hilbert term is proportional to H^{-2} and its derivative remains proportional to H^{-2}. These are power-law local responses, not undressed anomaly logarithms.

Each curvature-squared integral is independent of H on the round sphere. Its logarithmic derivative therefore vanishes. This includes finite R^2 shifts, the topological Euler counterterm and equivalent quadratic-curvature bases. The Weyl-squared term vanishes identically, and $\int \sqrt{g}\,\Box R = 0$ because R is constant and the manifold has no boundary. Thus no dimension-four local counterterm produces a non-zero contribution in the compact cohomological projection.

Higher-derivative local EFT operators beyond the four-derivative curvature-squared basis scale with non-zero powers of H on the round sphere and remain local power-law responses. Massive and running theories also contain non-local functions of ratios such as m_i/H and coupling-dependent threshold terms. These can be scheme-independent information in the full determinant, but they are not local undressed Wess–Zumino logarithms and are not converted into one by a finite local counterterm. The only non-zero constant in the non-trivial local Weyl cohomology class is the Euler coefficient a. □

Define the compact coefficient functional by

$$\text{Coeff}_{A,S^4}\left(\alpha\mathbf{e}_A^{S^4}\right) = \alpha. \tag{37}$$

The corresponding scalar compact extraction is therefore

$$\mathcal{E}_{S^4}[\Gamma] \equiv \frac{1}{4}\ \text{Coeff}_{A,S^4}(\mathcal{R}_{S^4}[\Gamma]) = a. \tag{38}$$

Loop order and cohomology. At one loop, the preceding uniqueness proof is the heat-kernel/local counterterm classification on the compact saddle. Beyond one loop, the corresponding statement is the Weyl-cohomology classification: local Weyl anomalies are constrained by Wess–Zumino consistency, and on conformally flat S^4 the only non-trivial local class capable of producing an undressed constant-Weyl logarithm is the Euler class. The cohomology protects the class and its scheme-independence. It does not fix the numerical value of a for an arbitrary interacting theory; that coefficient is the exact type-A coefficient of the theory under consideration, reducing to the free-field census only at the appropriate fixed point or ultraviolet limit.

The counterterm classification is displayed explicitly in Table 2; a basis-independent version is given in Appendix A.

Corollary 5 (Closed-saddle representative of the scale-response sector)**.** *The compact generator $\mathbf{e}_A^{S^4}$ is the closed-saddle representative of the paired generator $\mathbf{e}_A^{\text{dS}}$. The compact extraction $\mathcal{E}_{S^4}$ and the paired datum $\mathfrak{a}_{\text{dS}}$ therefore transmit the same unique QFT coordinate, a.*

Local term on $S^4(H)$	Scale behaviour	Response under $\mathrm{d}/\mathrm{d}\ln H$	Projected class
$\int\sqrt{g}$	H^{-4}	power law, $\propto H^{-4}$	zero
$\int\sqrt{g}\,R$	H^{-2}	power law, $\propto H^{-2}$	zero
$\int\sqrt{g}\,R^2$ and equivalent quadratic terms	H^0	zero	zero
$\int\sqrt{g}\,E_4$	topological constant	zero	zero
$\int\sqrt{g}\,C^2$	zero	zero	zero
$\int\sqrt{g}\,\Box R$	zero	zero	zero
$-4a\ln(\mu/H)$	logarithmic	$4a$	$4a$

Table 2: Local terms in the compact-S^4 constant-Weyl response. The finite Euler counterterm shifts only an additive constant in Γ; it cannot change the coefficient of the anomaly logarithm.

Proof. Theorem 2 identifies the Euler/Wess–Zumino class as the sole contribution to the local de Sitter scale-response sector. Theorem 4 shows that the compact constant-Weyl map extracts exactly its coefficient and no other local counterterm-invariant constant. The two maps therefore agree on the closed-saddle realisation. □

This corollary is the first part of the argument that the appearance of a is not peculiar to a specially constrained sphere calculation. A second, observer-centred realisation is supplied by the static-patch horizon.

4 Static patch: the horizon realisation

4.1 Regulated Euclidean geometry

The Euclidean static patch of four-dimensional de Sitter space is [28]

$$\mathrm{d}s_E^2 = f(r)\mathrm{d}\tau^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2\mathrm{d}\Omega_2^2, \qquad f(r) = 1 - H^2r^2, \qquad \tau \sim \tau + \frac{2\pi}{H}. \tag{39}$$

The Euclidean-time circle shrinks smoothly at $r = H^{-1}$, leaving the round horizon two-sphere as a bolt. Introduce a cutoff surface at $r = r_0 < H^{-1}$ and retain the horizon-side region

$$\mathcal{R}_{r_0} = \{r_0 \le r \le H^{-1}\}. \tag{40}$$

Its boundary is $S^1 \times S^2$ and its topology is $D^2 \times S^2$. This codimension-one worldtube is an auxiliary regulator of the codimension-two horizon cut; it is not postulated to be a physical boundary of the theory.

Near the bolt, proper distance ρ puts the metric in the form

$$\mathrm{d}s_E^2 = \mathrm{d}\rho^2 + \rho^2\mathrm{d}\theta^2 + H^{-2}\mathrm{d}\Omega_2^2 + O(\rho^2), \qquad \theta \sim \theta + 2\pi. \tag{41}$$

A cutoff at $\rho = \delta$ therefore has a contractible circle of radius $R_{S^1} \sim \delta$ and a transverse sphere of radius $R_{S^2} = H^{-1}$. The only independent dimensionless geometric modulus formed from the two symmetry-preserving radii is

$$\phi_T = \ln\left(\frac{R_{S^1}}{R_{S^2}}\right). \tag{42}$$

The coefficient of the logarithmic response to this ratio is the object to be compared with the compact-sphere extraction.

4.2 The boundary sector is present and must be tested

Conformal flatness removes the bulk type-B term and the Weyl–extrinsic invariant j_1, but it does not make the full boundary anomaly one-dimensional. The cutoff surface is not totally umbilic, so j_2 is generally non-zero. With the orientation and extrinsic curvature convention specified in Appendix B, its principal curvatures are

$$k_\tau = -\frac{H^2 r_0}{\sqrt{f_0}}, \qquad k_\Omega = \frac{\sqrt{f_0}}{r_0}, \qquad f_0 = 1 - H^2 r_0^2, \tag{43}$$

and hence

$$\operatorname{Tr}\widehat{K}^3 = -\frac{2}{9 r_0^3 f_0^{3/2}}. \tag{44}$$

After integration over $S^1 \times S^2$,

$$\int_{S^1\times S^2} \sqrt{h}\,\operatorname{Tr}\widehat{K}^3 = -\frac{16\pi^2}{9 H r_0 f_0}, \tag{45}$$

up to the overall orientation sign. As the cutoff approaches the horizon,

$$r_0 = H^{-1} - \frac{1}{2} H\delta^2 + O(\delta^4), \qquad f_0 = H^2\delta^2 + O(\delta^4), \tag{46}$$

so

$$\int \sqrt{h}\,\operatorname{Tr}\widehat{K}^3 = -\frac{16\pi^2}{9H^2\delta^2} + O(\delta^0). \tag{47}$$

The δ^{-2} behaviour is the cutoff dependence of the integrated geometrical coefficient multiplying the logarithmic boundary-anomaly charge q_2. It does not reclassify $q_2 j_2$ as a non-logarithmic anomaly term.

Equations (45)–(47) show that the $q_2 j_2$ sector is not being set to zero. It is genuine logarithmic anomaly data once a smooth boundary, boundary condition and cutoff surface are specified. Its value is nevertheless tied to the extrinsic geometry and boundary specification of the auxiliary worldtube, has no closed-S^4 representative, and has no independent datum in the original no-boundary $n = 1$ spherical-horizon logarithm. The quotient therefore retains the regulator-independent Euler/Wess–Zumino coefficient of the codimension-two horizon observable rather than the complete codimension-one boundary effective action. This does not discard physical edge sectors: stretched-horizon analyses show that completed gauge, p-form, gravitational and stringy boundary problems can contain genuine bulk and edge data [55, 56, 58, 57].

4.3 The Euler/Wess–Zumino horizon logarithm

For a spherical entangling surface in a four-dimensional CFT, the universal logarithmic contribution is fixed by the type-A anomaly. Its convention-independent coefficient has magnitude

$$\left|\operatorname{Coeff}_{\ln(R/\delta)} S_{\rm univ}\right| = 2a\,\chi(S^2) = 4a. \tag{48}$$

For example, the Casini–Huerta–Myers convention gives $S_{\rm univ} = -4a\ln(R/\delta)$ in four dimensions; reversing the logarithm or the effective-action convention reverses the sign. The coefficient $4a$ is the invariant content used below. This follows from the conformal mapping of the spherical problem and from the Euler/Wess–Zumino analysis of the anomaly, including its boundary completion [16, 17, 13]. Related horizon-entropy analyses connect logarithmic terms to type-A and type-B anomaly data in conformally flat and black-hole settings [32, 33]. Gauge fields require the appropriate gauge-invariant algebra or edge contribution; with that treatment the spherical coefficient agrees with the standard type-A anomaly normalisation [18, 19, 53, 55].

At the de Sitter bolt, the ambient Weyl tensor vanishes and the traceless codimension-two extrinsic curvatures of the round horizon sphere vanish. The non-Euler surface functional therefore drops out, leaving the type-A term. Let $W_Q^{(n)}$ denote the Euclidean effective action on the replicated horizon geometry and define the regulated horizon entropy functional by

$$\mathcal{S}_Q^{\mathrm{hor}} \equiv (n\partial_n - 1)W_Q^{(n)}\Big|_{n=1} . \tag{49}$$

Equivalently, for the round sphere this is the thermal entropy obtained by the conformal map to the de Sitter static patch. Thus the definition does not require the replica construction as a computational method; it only fixes which horizon functional is being projected.

Using $R_{S^1} \sim \delta$ and $R_{S^2} = R_H = H^{-1}$ from Eq. (41), the projected logarithmic entropy representative can be written

$$\mathcal{S}_{\mathrm{log}}^{(a)} = \varepsilon\, 4a \ln\!\left(\frac{R_{S^2}}{R_{S^1}}\right) + \text{constant} = -\varepsilon\, 4a\phi_T + \text{constant}, \qquad \varepsilon = \pm 1, \tag{50}$$

where ε records the convention for the orientation of the logarithm and for the entropy/effective-action replica transform. Define the local horizon scale response by

$$\mathcal{R}_Q^{\mathrm{hor}} \equiv -\mathcal{P}_{\mathrm{loc}}^{(4)} \frac{\partial \mathcal{S}_Q^{\mathrm{hor}}}{\partial \phi_T}, \qquad R_{S^1} \simeq \delta \text{ held fixed.} \tag{51}$$

The convention-independent statement is that its Euler coefficient has magnitude $4a$ and contains no second QFT coordinate. Let $\mathbf{e}_A^{\mathrm{hor}}$ denote the horizon representative of the unit-normalised paired Euler/Wess–Zumino generator $\mathbf{e}_A^{\mathrm{dS}}$.

The equality

$$\chi(D^2 \times S^2) = \chi(S^4) = 2 \tag{52}$$

shows that the Euler class itself is represented consistently in the closed and tube geometries once the Chern–Gauss–Bonnet boundary term is included. Equation (48), rather than the Euler characteristic alone, fixes the universal horizon-entanglement coefficient. The topology and the spherical logarithm therefore play complementary roles: the first identifies the common Euler sector, and the second fixes its observer-centred logarithmic response.

Proposition 6 (Static-patch realisation)**.** *Under the regulator prescription in which the codimension-one worldtube regulates the $n = 1$ round de Sitter horizon cut, the regulator-independent local logarithmic response has a one-dimensional QFT image and is controlled by a. Modulo cohomologically trivial and auxiliary horizon-tube responses,*

$$\mathcal{R}_Q^{\mathrm{hor}} = \varepsilon\, 4a(Q)\mathbf{e}_A^{\mathrm{hor}}, \qquad \varepsilon = \pm 1. \tag{53}$$

Equivalently, a representative of the corresponding projected horizon-entropy functional is

$$\mathcal{S}_{\mathrm{log,proj}}^{\mathrm{hor}} = \varepsilon\, 4a(Q) \ln\!\left(\frac{R_{S^2}}{R_{S^1}}\right) + \text{const.} \tag{54}$$

The non-Euler boundary charge q_2 belongs to the full worldtube effective action, but not to the common universal scale-response sector.

Proof. The bulk type-B and j_1 sectors vanish by conformal flatness. The j_2 sector is non-zero but belongs to the auxiliary boundary problem and fails the closed-saddle and regulator-independence requirements, as shown explicitly in Eqs. (45)–(47). Counterterm-trivial sectors are removed by definition. Differentiating Eq. (50) with respect to ϕ_T, with the proper cutoff radius held fixed, gives Eq. (53). The corresponding horizon-entropy representative is Eq. (54), whose sole QFT coefficient is a. □

Theorems 2 and 4, together with Proposition 6, give the promised cross-realisation result:

$$\boxed{\text{closed } S^4 \text{ response} \quad \longleftrightarrow \quad a \quad \longleftrightarrow \quad \text{static-patch horizon logarithm.}} \tag{55}$$

The boxed statement equates only the normalised local anomaly coordinate, not the complete closed-sphere and static-patch partition functions. Full bulk-plus-edge analyses remain complementary: they describe the completed horizon Hilbert space or boundary problem, whereas the theorem identifies the common cohomological residue after auxiliary boundary data are removed [46, 20, 51, 53, 55, 56, 47, 58, 57, 48, 49, 34, 35, 50, 36].

5 The Standard Model value of the selected coordinate

5.1 Free-field anomaly census

The type-A anomaly coefficient is fixed by the microscopic QFT, not by the de Sitter geometry. For free four-dimensional fields in the standard normalisation [21, 22, 23],

Field	Type-A coefficient
Real scalar	1/360
Weyl fermion	11/720
Gauge vector	31/180

Equivalently,

$$720a = 2N_0 + 11N_{1/2} + 124N_1. \tag{56}$$

The unbroken minimal Standard Model contains four real Higgs components, forty-five Weyl fermions and twelve gauge vectors:

$$(N_0, N_{1/2}, N_1) = (4, 45, 12). \tag{57}$$

Thus

$$a_{\mathrm{SM}} = \frac{4}{360} + \frac{45 \times 11}{720} + \frac{12 \times 31}{180} \tag{58}$$

$$= \frac{1991}{720} \simeq 2.7653. \tag{59}$$

Equation (59) is a spin-weighted field-content census. It is not a sum of particle masses, a fit to galactic or cosmological data, or a number generated by the radius or topology of S^4. The geometry selects the Euler coordinate; the QFT supplies its value. The same number therefore appears in the compact and horizon realisations for the same reason that the same central charge appears in different observables of a CFT: they probe the same anomaly class.

5.2 Interacting theories and the meaning of a_{SM}

At an interacting conformal fixed point, a is the coefficient of the non-trivial Euler anomaly class of that fixed-point theory. Along an RG flow its fixed-point values are subject to the four-dimensional a-theorem [24]. In locally covariant interacting QFT, Wess–Zumino consistency and finite-renormalisation analysis continue to distinguish the non-trivial anomaly classes from cohomologically removable terms [25].

The minimal Standard Model is not an exact conformal field theory at all scales. Equation (59) therefore has the following precise scope: it is the standard free-field, unbroken Standard Model anomaly census used when the microscopic channel is evaluated from that field content. Thresholds, symmetry breaking and running couplings enter the full effective action and can affect which fixed-point or effective value of a is appropriate to a particular UV completion. They do not alter the one-dimensionality theorem. Once the microscopic theory is specified, the universal local de Sitter channel transmits its value of a and no independent second coordinate.

This distinction is useful phenomenologically. New particle content changes the free-field census by a known representation-theoretic amount. For example, adding ΔN_0 real scalars, $\Delta N_{1/2}$ Weyl fermions and ΔN_1 vectors gives

$$\Delta a = \frac{2\Delta N_0 + 11\Delta N_{1/2} + 124\Delta N_1}{720}. \tag{60}$$

A de Sitter mechanism genuinely controlled by this channel is therefore sensitive to the anomaly census of the microscopic field content, but blind within the channel to masses, Yukawa matrices and gauge couplings as separate inputs.

Corollary 7 (Status of a_{SM})**.** *Under the unbroken minimal Standard Model free-field evaluation of the microscopic anomaly coefficient, the unique QFT coordinate selected by the universal local de Sitter scale-response sector is*

$$\mathfrak{a}_{\mathrm{dS}}(Q_{\mathrm{SM}}) = a_{\mathrm{SM}} = \frac{1991}{720}. \tag{61}$$

Its occurrence is not specific to the compact-S^4 extraction: the same coordinate controls the static-patch horizon logarithm.

This is the sense in which a_{SM} is a real matter datum of the channel. The claim is neither that every gravitational observable depends only on a_{SM} nor that a_{SM} alone fixes a dimensionful observable. It is that two geometrically distinct representatives of the universal local de Sitter scale response transmit the same protected QFT coordinate, whose minimal Standard Model value is Eq. (59).

6 Application to cosmological-constant channels

The result separates three statements that are often conflated. First, the renormalised cosmological constant is the physical coupling multiplying the volume term in the gravitational effective action. Second, on a single compact saddle its split into an "absolute vacuum energy" and a cosmological counterterm is not invariant: both occupy the same local operator. Third, the projected local logarithmic de Sitter response does contain a protected matter datum, uniquely identified here as a.

A construction whose microscopic dependence genuinely factors through this channel therefore has the matter-side form

$$\mathcal{O}_{\mathrm{dS}} = \mathcal{F}[g, H, \ldots; a(Q)], \tag{62}$$

with no independent microscopic coordinate from the excluded sectors. If the construction is linear in the projected anomaly action, this reduces to

$$\mathcal{O}_{\mathrm{dS}}^{\mathrm{lin}} = F[g, H, \ldots]a(Q), \tag{63}$$

not to a term proportional to a separately invariant absolute density. In the minimal Standard Model free-field evaluation, the coordinate in Eq. (62) is $a_{\mathrm{SM}} = 1991/720$.

Corollary 8 (Matter datum for compact de Sitter Λ channels)**.** *Any calculation that reads a universal local matter contribution to Λ through the compact/de Sitter scale-response channel must use the type-A anomaly coefficient as its QFT datum. It cannot depend independently, within the same channel, on an absolute zero-point-energy density, c, q_1, q_2, particle masses,* *Yukawa couplings or local counterterm conventions. For the unbroken minimal Standard Model free-field census,*

$$a_{\mathrm{SM}} = \frac{1991}{720}. \tag{64}$$

The corollary fixes the matter coordinate, not the value of Λ. Because a is dimensionless whereas Λ is a curvature scale, a gravity-side normalisation, matching condition or scale-setting principle is still required.

This classification is independent of which gravity-side dynamics is chosen. Anomaly-induced actions, non-local constraints, four-form sectors or global sequestering mechanisms may use anomaly data in different ways [39, 40, 41, 42, 43, 44]; the present theorem says only that a construction restricted to this local de Sitter channel has no competing matter coordinate.

The scale-setting caveat is essential. A complete mechanism must specify the gravitational saddle, normalisation, dimensionful scale and dynamics. Radiative stability is also separate: vacuum-energy differences between physical phases, including electroweak and QCD phases, are meaningful relative quantities and remain genuine dynamical data. The theorem removes an ill-defined absolute zero-point sum from the list of independent single-saddle matter inputs; it does not remove the physical problem posed by phase-dependent energy differences.

The resulting cosmological task is therefore

> Determine the gravitational scale-setting principle by which the protected anomaly coordinate a combines with geometric data to give the renormalised cosmological constant.

For the Standard Model census this is a matching problem involving a_{SM}, rather than a cancellation problem formulated in terms of a separately invariant M_P^4 matter density.

7 Discussion

7.1 What has been proved

The paper establishes a theorem chain.

1. The isotropic local logarithmic scale response of exact round de Sitter, represented by the closed S^4 saddle and the $n = 1$ round static-patch horizon logarithm, has one-dimensional QFT image once finite local counterterms, cohomologically trivial Weyl cocycles and auxiliary boundary data are quotiented out.

2. The surviving sector is the Euler/Wess–Zumino class and its single QFT coordinate is a.

3. The compact-S^4 constant-Weyl extraction gives exactly this coordinate and no competing local counterterm-invariant constant.

4. The universal spherical horizon logarithm gives the same coordinate even though the complete regulated boundary problem contains additional $q_2 j_2$ data.

5. Evaluated with the unbroken minimal Standard Model free-field content, the coordinate is $a_{\mathrm{SM}} = 1991/720$.

The first four statements together constitute the new cross-realisation result. Their individual ingredients—the anomaly basis, Chern–Gauss–Bonnet theorem, boundary charges, spherical $4a$ logarithm and compact renormalisation ambiguity—are standard. The new content is their common-image classification and the explicit non-zero boundary-sector test: requiring the closed sphere and round horizon to represent the same universal local response removes every non-Euler coordinate for a distinct reason. This upgrades the compact observation into a cross-realisation selection theorem.

7.2 Scope

The theorem is local and sector-specific: it classifies the isotropic local logarithmic de Sitter scale response, not the complete sphere partition function, static-patch density matrix, non-local determinant, higher stress-tensor data, Rényi defect coefficients or boundary observable algebra. Physical boundary CFTs retain their boundary charges q_1 and q_2. Those charges are genuine when the boundary and its conditions are part of the physical specification; they are projected out here because the worldtube is auxiliary and because no corresponding boundary datum exists on the closed saddle.

The theorem is also tied to conformally flat de Sitter geometry and a round horizon cut. On a non-conformally-flat Einstein manifold the c-anomaly can survive. For a non-spherical entangling surface, extrinsic type-B surface functionals can contribute. They probe larger channels with more microscopic coordinates, exactly as expected.

Finally, $a_{\text{SM}} = 1991/720$ is the free-field unbroken Standard Model census. A specified interacting fixed point or UV completion carries its own value of a. The structural result is the selection of one coordinate; the minimal Standard Model census is the corresponding free-field evaluation.

7.3 Falsifiability of the channel claim

The selection theorem gives a direct diagnostic. Suppose a proposed observable is said to arise solely from the universal local de Sitter anomaly channel. If its microscopic formula depends independently on c, on a boundary charge, on a particle mass or on a Yukawa coupling, then at least one of two things is true: either the observable draws on additional sectors of the effective action, or the claimed channel identification is incorrect. Conversely, changing the microscopic field content while keeping a fixed cannot change a genuinely channel-only observable.

This is a useful selection result. It restricts which microscopic data can consistently appear in anomaly-channel models and distinguishes such models from general effective-action constructions.

8 Conclusion

The universal local logarithmic anomaly channel of exact round de Sitter has a single QFT coordinate: the type-A anomaly coefficient. On the closed S^4 saddle this coordinate is isolated by a counterterm-invariant constant-Weyl extraction. On the regulated static patch it controls the universal spherical horizon logarithm after genuine but auxiliary boundary-anomaly data are separated from the common channel. The agreement of these two realisations shows that the selected coordinate is not an artefact of a single highly constrained compact geometry.

For the unbroken minimal Standard Model free-field content, this coordinate is

$$a_{\text{SM}} = \frac{1991}{720}.$$

The number is therefore best understood as the Standard Model value of the universal local de Sitter scale-response datum: the coefficient of the Euler/Wess–Zumino anomaly class. Any gravitational construction whose microscopic dependence genuinely factors through this channel inherits that datum and no independent mass, coupling, type-B or boundary-charge input.

The theorem identifies the protected matter coordinate that a channel-based determination of Λ must use. The remaining problem is to derive the gravitational scale and matching principle that convert this dimensionless QFT datum into a physical cosmological constant.

A Counterterm classification on round S^4

This appendix records the basis-independent content of the uniqueness step in Theorem 4. A maximally symmetric four-sphere obeys

$$R_{\mu\nu} = 3H^2 g_{\mu\nu}, \qquad R_{\mu\nu\rho\sigma} = H^2(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}). \tag{65}$$

Consequently,

$$\int \sqrt{g} = \frac{8\pi^2}{3H^4}, \tag{66}$$

$$\int \sqrt{g}\, R = \frac{32\pi^2}{H^2}, \tag{67}$$

$$\int \sqrt{g}\, R^2 = 384\pi^2, \tag{68}$$

$$\int \sqrt{g}\, R_{\mu\nu}R^{\mu\nu} = 96\pi^2, \tag{69}$$

$$\int \sqrt{g}\, R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = 64\pi^2, \tag{70}$$

$$\int \sqrt{g}\, E_4 = 64\pi^2. \tag{71}$$

The Weyl tensor vanishes and $\Box R = 0$ pointwise.

Every quadratic-curvature basis can be reduced to $\{E_4, C^2, R^2, \Box R\}$. The first and third integrals are constants on the round-sphere family and therefore have zero derivative with respect to $\ln H$; the second and fourth vanish. Finite shifts of the Euler coefficient in the local gravitational action change the additive constant in the sphere effective action but not the coefficient of $\ln(\mu/H)$.

A generic higher-curvature local monomial with $2n$ derivatives scales as

$$\int_{S^4(H)} \sqrt{g}\, \mathcal{R}^n \propto H^{2n-4}, \tag{72}$$

where $\mathcal{R}^n$ denotes a scalar made from n curvature tensors and covariant derivatives evaluated on the maximally symmetric background. For $n \neq 2$ its scale response is a non-zero power of H; for $n = 2$ it is constant. Neither case produces an undressed logarithm. This is why extending the local EFT basis does not add a second constant to the projected Wess–Zumino response.

The one subtlety is that the full determinant of a massive theory can contain terms such as $F(m/H)$ or logarithms dressed by powers of m/H. Such terms can be physical and scheme-independent. They are not members of the local anomaly sector isolated by $\mathcal{P}^{(4)}_{\rm loc}$ and the subsequent cohomological quotient. The compact theorem classifies the local protected constant, not the entire determinant.

B Extrinsic boundary invariant on the static-patch tube

For the metric (39), take the horizon-side region $r_0 \le r \le H^{-1}$ and the outward unit normal on its boundary at $r = r_0$ to point toward decreasing r:

$$n^\mu = -\sqrt{f_0}\left(\frac{\partial}{\partial r}\right)^\mu. \tag{73}$$

Use the convention

$$K_{ab} = -h_a{}^\mu h_b{}^\nu \nabla_\mu n_\nu. \tag{74}$$

The mixed principal curvatures of the induced $S^1 \times S^2$ geometry are then

$$k_\tau = -\frac{H^2 r_0}{\sqrt{f_0}}, \qquad k_\theta = k_\phi = k_\Omega = \frac{\sqrt{f_0}}{r_0}. \tag{75}$$

Writing $\Delta = k_\tau - k_\Omega$, the traceless eigenvalues are $2\Delta/3, -\Delta/3, -\Delta/3$, so

$$\operatorname{Tr} \widehat{K}^3 = \frac{2}{9}\Delta^3. \tag{76}$$

Because

$$\Delta = -\frac{H^2 r_0^2 + f_0}{r_0\sqrt{f_0}} = -\frac{1}{r_0\sqrt{f_0}}, \tag{77}$$

we obtain Eq. (44).

The induced volume element is

$$\sqrt{h} = \sqrt{f_0}\, r_0^2 \sin\theta, \tag{78}$$

and the Euclidean-time period is $2\pi/H$. Therefore

$$\int_{S^1\times S^2} \sqrt{h}\operatorname{Tr}\widehat{K}^3 = \frac{2\pi}{H}(4\pi)\sqrt{f_0} r_0^2 \left(-\frac{2}{9 r_0^3 f_0^{3/2}}\right) \tag{79}$$

$$= -\frac{16\pi^2}{9 H r_0 f_0}. \tag{80}$$

The opposite convention for K_{ab} or the opposite orientation flips the sign but not the cutoff dependence or sector assignment.

To relate r_0 to proper distance δ from the bolt, expand

$$\delta = \int_{r_0}^{H^{-1}} \frac{\mathrm{d}r}{\sqrt{1 - H^2 r^2}}. \tag{81}$$

This gives

$$H^{-1} - r_0 = \frac{1}{2} H\delta^2 + O(\delta^4), \qquad f_0 = H^2\delta^2 + O(\delta^4), \tag{82}$$

and hence Eq. (47). The calculation proves that the j_2 invariant is present in the full regulated boundary problem and depends explicitly on the auxiliary cutoff geometry. The conclusion does not rely on subtracting only the leading divergence; the point is that j_2 is a datum of the auxiliary codimension-one boundary problem, not of the original no-boundary horizon observable. Its projection out of the common channel is therefore substantive rather than automatic.

C Normalisation of the Standard Model coefficient

For completeness, the field count in Eq. (57) is as follows. The Higgs doublet contains four real scalar components. Each generation has fifteen Weyl fermions when no right-handed neutrino is included: six quark-doublet components, three up-type singlets, three down-type singlets, two lepton-doublet components and one charged lepton singlet. Three generations therefore give forty-five Weyl fermions. The gauge sector contains eight gluons, three weak vectors and one hypercharge vector, for a total of twelve.

Substitution into Eq. (56) gives

$$720 a_{\mathrm{SM}} = 2(4) + 11(45) + 124(12) = 8 + 495 + 1488 = 1991. \tag{83}$$

If right-handed neutrinos or any other beyond-Standard-Model fields are included, the count changes according to Eq. (60).

Acknowledgements

The author declares no competing financial interests. This research received no external funding.

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