A compact S^4 anomaly channel for the cosmological constant

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Abstract

The cosmological-constant problem is usually framed as a sum of absolute vacuum energies. On compact conformally flat S^4, however, the vacuum functional is degenerate with the cosmological counterterm. We prove that, after finite local counterterm directions are quotiented out, the type-A Euler anomaly coefficient is the unique nonvanishing scheme-independent datum in the local linear constant-Weyl response. For the minimal Standard Model its leading free-field value is

$$a_{\mathrm{SM}} = \frac{1991}{720}.$$

The corresponding local metric variation produces an H^4 stress and no small-curvature branch, so additional infrared and gravitational structure is required.

The round sphere also carries an intrinsic degree-one chiral-spin class. Under the QCD matching and compact-gravity prescription developed below, this supplies the proton-scale factor m_p/M_P and the compact gravitational factor $e^{-24\pi^2}$. The resulting leading value and the Planck 2018 flat-ΛCDM inference are

$$\begin{aligned}\left(\frac{\Lambda_\infty}{M_P^2}\right)_{\mathrm{calc}} &= \frac{1991}{720}\frac{m_p}{M_P}e^{-24\pi^2} = 2.856 \times 10^{-122},\\ \left(\frac{\Lambda}{M_P^2}\right)_{\mathrm{obs}} &= (2.846 \pm 0.057) \times 10^{-122},\end{aligned}$$

a 0.4% difference in the central values. A proposed four-form completion maps the compact response to the residual cosmological curvature and gives $w = -1$ under the stated late-time condition. No continuous parameter is fitted to the cosmological value.

Keywords: cosmological constant, compact S^4, conformal anomaly, chiral spin bundle, QCD baryon sector, semiclassical gravity

1 Introduction

The observed dark-energy density is smaller than the Planck density by roughly 120 orders of magnitude [1, 2, 3, 5, 6, 7, 8]. The usual formulation asks how large zero-point contributions can cancel to leave such a small residual. The ambiguity is not created by compactness: even in flat space the absolute vacuum-energy density is not a scheme-independent observable. On round S^4 without boundary the situation is sharper still, because the homogeneous vacuum term and the cosmological counterterm are the same local functional, $\int \sqrt{g}\, d^4x$. Before one can ask whether the

Standard Model predicts Λ, one must identify which compact-S^4 datum is protected against this renormalisation ambiguity.

We first show that, on conformally flat S^4, the type-A Euler anomaly coefficient is the unique nonvanishing scheme-independent datum in the local linear constant-Weyl response. This result requires no infrared scale, de Sitter contour or cosmological source model; it is a theorem about the compact representative itself. For the minimal Standard Model its leading free-field value is

$$\boxed{a_{\rm SM} = \frac{1991}{720} = 2.76528.}$$

We then ask a more ambitious question: can this protected compact datum enter a complete dimensionless response? The round spin sphere contains an intrinsic degree-one chiral-spin clutching class. A single matter-side premise identifies its homotopy class with the primitive QCD chiral class at the infrared matching slice; the resulting $B = 1$ QCD+QED sector has proton spectral floor m_p. A separate compact gravitational calculation supplies the classical saddle factor $e^{-24\pi^2}$ in the $u = 1$ decaying sector. These ingredients give the leading compact-channel value

$$q_{\rm comp} = a_{\rm eff}\frac{m_p}{M_P}e^{-24\pi^2},$$

with $a_{\rm eff} \to a_{\rm SM}$ at leading Standard Model order. We then introduce a four-form global-source completion that maps this rigid compact response to the residual scalar-curvature mode. Several of these structural choices have independent precedents in the cosmological-constant literature; Sec. 11 separates that convergence from the ingredients that remain specific to the present construction.

The paper is organized in that order. Sections 2–4 contain the counterterm analysis, extraction theorem and Standard Model evaluation. Section 5 develops the infrared matter bridge; Sec. 6 gives the compact gravitational weight; and Sec. 7 states the resulting compact benchmark. Only after that result is assembled do Secs. 8 and 9 address the separate question of how such a response could source curvature. Section 7 exposes the compact-channel status ledger at the point of assembly; Sec. 10 gives the quantitative correction budget and observational tests.

Framework and scope. We work at leading saddle order in a selected compact de Sitter sector. Standard Model matter is treated quantum mechanically on the compact background, while the gravitational weight is the classical Einstein–Hilbert saddle factor. The protected coefficient is the matter-QFT Euler anomaly. Fixed curvature-squared/Euler terms and absolute determinant constants are scheme dependent; source-dependent gravitational dressing and higher-curvature changes to the compact action enter the correction budget of Sec. 10.

Conventions. We use $\kappa^2 = 8\pi G$ and the unreduced Newton mass $M_P = G^{-1/2} = 1.221 \times 10^{19}$ GeV [4]. The compact saddle curvature is $\Lambda_{\rm UV}$, with $u \equiv \kappa^2\Lambda_{\rm UV}$. The late-time curvature supplied by the proposed global-source completion is Λ_∞; under its stated late-time condition, $q = G\Lambda_\infty = \Lambda_\infty/M_P^2$ on the numerical Newton branch. We denote the observational comparison by $\Lambda_{\rm obs}$. The compact input u and the output q refer to distinct curvatures. We work in Euclidean signature on round S^4; in the compact gravitational sector $R_{\mu\nu} = \Lambda_{\rm UV}g_{\mu\nu}$ and $R = 4\Lambda_{\rm UV} > 0$. The Euler density is $E_4 = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$ and $\int_{S^4}\sqrt{g}\,E_4 = 64\pi^2$ [21]. The trace-anomaly convention is $\langle T^\mu{}_\mu\rangle = (c/16\pi^2)W^2 - (a/16\pi^2)E_4 +$total derivatives [63, 18, 19], and $S_{\rm EH} = -(1/2\kappa^2)\int(R - 2\Lambda_{\rm UV})\sqrt{g}\,d^4x$.

2 The compact-S^4 vacuum-energy ambiguity

The cosmological counterterm $\alpha\int\sqrt{g}$ in the renormalised effective action of any QFT on a curved background is a familiar feature of semiclassical gravity [9, 13, 14, 7]. In flat space, the absolute vacuum-energy density is not a scheme-independent observable: a hard cutoff gives the familiar quartic dependence on the cutoff, while dimensional regularisation gives a different mass-dependent contribution. What flat space does provide is a reference structure in which subtraction conventions can be imposed, and in which *differences* between configurations (Casimir-type energies) can be physically meaningful. On compact S^4 without boundary the situation is structurally sharper. There is a single compact configuration, and the vacuum-energy functional and the cosmological-counterterm functional are both scheme-dependent and the *same* local operator on S^4. Both reduce to $\int\sqrt{g}\,d^4x \propto H^{-4}$, with no geometrically distinguishable label separating them; no renormalisation condition intrinsic to the compact background splits the two. The observation that the standard zero-point sum simply renormalises the bare cosmological constant, and that the split between "vacuum energy" and "counterterm" is not a scheme-independent observable, has been made by Bianchi and Rovelli [16] and by Hollands and Wald [17]; related recent discussions also separate observable QFT effects from a literal absolute vacuum-energy fluid [67]. The absolute vacuum energy on S^4 is therefore not a scheme-independent observable. Any well-posed extraction must identify a counterterm-invariant functional.

Why Casimir observables are different. The Casimir effect measures an energy *difference* between physically distinct configurations, for example parallel conducting plates at separation d versus $d \to \infty$ [20]. The common cosmological counterterm cancels in that subtraction. On compact S^4 without boundary there is no intrinsic second configuration: the partition function is a single-saddle quantity, so the local volume ambiguity does not cancel. Renormalized differences between genuinely distinct configurations or phases can likewise be observable; that does not supply a separately invariant absolute vacuum-energy datum on the single compact saddle.

Three tempting direct readings of the compact path integral fail for the same reason. A free-energy density, the finite value of $\Gamma[S^4]$, or the absolute normalization of $Z[S^4]$ each retains local-counterterm or normalization dependence. None therefore defines the protected matter datum sought here. The construction below instead extracts the coefficient of the surviving Weyl-cohomology class.

A well-posed extraction must therefore be invariant under every allowed renormalisation ambiguity. The next section implements that quotient and asks which local constant-Weyl datum survives.

3 Extraction theorem on conformally flat S^4

$\Gamma[g]$ is defined only up to local counterterms. We therefore specify minimal consistency requirements for extracting a physical compact matter datum relevant to a cosmological channel from the path integral on S^4.

Local counterterm ambiguities. For any QFT on a curved background, renormalisation permits adding local curvature counterterms to $\Gamma[g]$:

$$\begin{aligned}\Gamma[g] \;\to\; \Gamma[g] &+ \alpha\int d^4x\sqrt{g} + \beta\int d^4x\sqrt{g}\,R \\ &+ \gamma\int d^4x\sqrt{g}\,R^2 + \delta\int d^4x\sqrt{g}\,W^2 + \epsilon\int d^4x\sqrt{g}\,E_4 + \cdots\,,\end{aligned} \tag{1}$$

together with scheme-dependent total derivatives such as $\int\sqrt{g}\,\Box R$. This is the local gravitational counterterm basis through four derivatives (modulo the Lanczos identity expressing $\int\sqrt{g}\,R_{\mu\nu}R^{\mu\nu}$ as a linear combination of the above) [9, 14]. The $\alpha\int\sqrt{g}$ term is the cosmological counterterm; shifting the matter Lagrangian by a constant $\mathcal{L}_m \to \mathcal{L}_m + \rho_0$ is equivalent to $\alpha \to \alpha + \rho_0$. Any putative extraction of a physical Λ from Z must be insensitive to these shifts; otherwise $Z \to \Lambda$ is not a prediction of the compact path integral but an artefact of the regularisation.

Extraction criteria. We require the extraction functional $\mathcal{F}[\Gamma; S^4(H)]$ to satisfy:

C1 (Counterterm/scheme invariance): $\mathcal{F}$ is invariant under all local counterterm ambiguities of Eq. (1), including $\alpha\int\sqrt{g}$ (cosmological counterterm), $\beta\int\sqrt{g}\,R$, higher-curvature terms, and total-derivative ambiguities, so that $\mathcal{F}$ constitutes a well-posed prediction of the path integral.

C2 (Conformally-flat universality): on round S^4 ($W^2 = 0$), the extracted quantity depends only on diffeomorphism-invariant, scheme-independent universal data of the matter sector, not on gauge-volume normalisations, measure conventions, or regulator artefacts.

Theorem 1 (Euler extraction on conformally flat S^4)**.** *On round conformally flat $S^4(H)$, among local linear constant-Weyl responses of the renormalised effective action satisfying counterterm invariance (C1) and conformal-flatness universality (C2), the unique nonvanishing H-independent scheme-independent matter datum is the type-A Euler anomaly coefficient. With $\Gamma(H) = -\ln Z[S^4(H)]$ and*

$$\langle T_{\mu\nu}\rangle = \frac{2}{\sqrt{g}}\frac{\delta\Gamma}{\delta g^{\mu\nu}},$$

it is isolated at a conformal fixed point by

$$\mathcal{A}[\Gamma] = -\frac{1}{4}\Pi_{H^0}\left(\frac{d\Gamma(H)}{d\ln H}\right). \tag{2}$$

Here Π_{H^0} extracts the coefficient of the H^0 term in the local constant-Weyl response after beta-function and operator-mixing contributions have been separated. Away from a fixed point, the same statement applies to the Euler-cocycle projection of the local RG equation after beta-function and operator-mixing terms have been separated [26].

Proof. The local counterterms in Eq. (1) evaluate on $S^4(H)$ as H^{-4}, H^{-2}, a constant, zero, a topological constant, and zero for the volume, Einstein, R^2, W^2, E_4, and $\Box R$ terms, respectively. After $d/d\ln H$, they are therefore either H-dependent or vanish. A finite E_4 counterterm changes only the constant part of Γ and cannot absorb the anomalous logarithm.

The sign is fixed by the Euclidean variational convention. Under a constant Weyl variation $g_{\mu\nu} \to e^{2\sigma}g_{\mu\nu}$, $\delta g^{\mu\nu} = -2\sigma g^{\mu\nu}$, so

$$\begin{aligned}\delta_\sigma\Gamma[g] &= -\int d^4x\sqrt{g}\,\sigma\langle T^\mu{}_\mu\rangle \\ &= +\frac{a}{(4\pi)^2}\int d^4x\sqrt{g}\,\sigma E_4\end{aligned} \tag{3}$$

for the anomaly convention of Eq. (4). On conformally flat S^4, $W^2 = 0$, the integrated total derivative vanishes, and $\int_{S^4}\sqrt{g}\,E_4 = 64\pi^2$. Hence $\delta_\sigma\Gamma = 4a\sigma$. Since a Weyl enlargement of the sphere sends $H \to e^{-\sigma}H$, one has $\sigma = -\delta\ln H$, and therefore

$$\Pi_{H^0}\left(\frac{d\Gamma}{d\ln H}\right) = -4a.$$

Equation (2) follows. Wess–Zumino consistency protects the coefficient [25]. At a fixed point the argument is loop-order independent: a local curvature term of order n scales as H^{2n-4}, while the only undressed logarithmic response in the conformally flat local cohomology is the Euler cocycle. For a running theory the local-RG Euler projector removes the additional $\beta^i \partial\Gamma/\partial g^i$ and operator-mixing responses. $\square$

Free-scalar sign audit. For one real conformally coupled scalar, $a_s = 1/360$ and the conformal Laplacian on S^4 has $\zeta_{\Delta_c}(0) = -1/90$ [11]. Because its eigenvalues scale as H^2,

$$\Gamma_s = \frac{1}{2} \ln \det(\Delta_c/\mu^2) = \frac{1}{90} \ln \frac{\mu}{H} + \text{constant}, \qquad \frac{d\Gamma_s}{d \ln H} = -\frac{1}{90} = -4a_s.$$

This determinant check fixes the sign independently of the Ward-identity argument and agrees with the standard even-dimensional sphere-free-energy convention [69].

Scope of the theorem. The theorem fixes the one-dimensional protected matter direction entering the connected compact response: it is the Euler coefficient a. It does not state that an ordinary metric variation of this datum produces the Friedmann cosmological constant; Sec. 8 shows that the corresponding local stress scales as H^4. The gravity-side map is therefore a separate ingredient.

Relation to the standard trace anomaly. The trace anomaly and Gauss–Bonnet integral are standard. The counterterm-by-counterterm classification on compact round S^4 sharpens this result: every allowed local ambiguity has either an H-dependent response or zero response under $d/d \ln H$, leaving the anomalous logarithm as the unique nonzero constant term.

Interpretation of the extracted datum. On $S^4(H)$, the protected coefficient a_{SM} appears in the anomaly-induced logarithm $+4a_{\text{SM}} \ln(\mu/H)$, whose constant-Weyl response is the H-independent term $-4a_{\text{SM}}$. This logarithm is the round-S^4 reduction of the anomaly-induced Riegert/Wess–Zumino effective action for the four-dimensional trace anomaly [33, 34, 35, 51, 68, 27]. The extraction map in Eq. (2) projects onto its Euler coefficient. The remaining effective action may contain local counterterm classes, beta-function/operator responses, and a Weyl-invariant complement; the projector excludes these additional structures without asserting that they vanish.

4 Standard Model evaluation of the extracted datum

In four dimensions the trace of the stress tensor contains the Euler and Weyl central charges [18, 19]:

$$\langle T^\mu{}_\mu \rangle = \frac{c}{16\pi^2} W_{\mu\nu\rho\sigma} W^{\mu\nu\rho\sigma} - \frac{a}{16\pi^2} E_4 + (\text{total derivatives}). \tag{4}$$

On round S^4, $W_{\mu\nu\rho\sigma} = 0$, so the type-B term drops out. The extraction theorem of Sec. 3 projects onto the Euler coefficient and removes the local counterterm classes.

At a free conformal fixed point the standard coefficients are [10, 11, 36, 37]

Field type	a
Real scalar	1/360
Weyl fermion	11/720
Vector boson	31/180

The minimal Standard Model contains four real Higgs scalars, 45 Weyl fermions and 12 vector bosons [4]. Hence

$$\boxed{a_{\rm SM} = \frac{4}{360} + \frac{45\times 11}{720} + \frac{12\times 31}{180} = \frac{1991}{720} = 2.76528.} \tag{5}$$

This rational number is the leading free-field Standard Model datum, fixed entirely by field content.

The full Standard Model is interacting and not exactly conformal. We therefore distinguish the protected coordinate a from the approximation used to evaluate it. We write $a_{\rm eff}$ for the interacting curved-space Euler coefficient appropriate to the microscopic compact matter channel, and use $a_{\rm SM}$ as its leading free-field value. Local-RG analyses separate the Euler coefficient from beta-function-proportional flow terms [26, 38]. In the explicit locally covariant ϕ^4 calculation of Fröb and Zahn, the effective second-order interaction contribution to the Euler coefficient vanishes [50]. This provides a structural check at the order calculated; the full curved-space Standard Model value remains to be calculated.

The scalar field-count contribution is also independent of the Higgs nonminimal curvature coupling $\xi|H|^2R$. Away from conformal coupling the full scalar trace can contain additional nonconformal local structures, but the type-A Euler and type-B Weyl coefficients themselves are independent of ξ [10, 9, 12]. On round S^4, $\int\sqrt{g}\,R^2$ is independent of the sphere scale and is annihilated by the extraction derivative, while $\int\sqrt{g}\,\Box R = 0$. The extracted Euler coordinate therefore does not require the assumption $\xi = 1/6$.

The extracted Euler coefficient is dimensionless and order unity. A dimensionless cosmological response therefore requires an independent infrared-to-Planck hierarchy. This hierarchy is not generated by the local analytic mass dependence of the same compact determinant: the heat-kernel expansion begins in even powers of the dimensionless mass [15], while perturbative Standard Model running gives $\alpha_s(M_P) \simeq 0.02$ [46], for which the Yang–Mills instanton weight $\exp[-2\pi/\alpha_s]$ is of order 10^{-136} [45]. Neither route supplies the required linear infrared-to-Planck factor. The construction therefore uses a distinct infrared readout, developed next from a physical QCD+QED sector tied to the intrinsic chiral-spin clutching class.

5 Intrinsic chiral-spin clutching and the QCD infrared response

The infrared factor is obtained from a QCD sector tied to the intrinsic spin geometry by one explicit class-level premise. The round spin geometry itself supplies the primitive integer and its degree-one boundary representative. This section derives that integer, states the QCD boundary premise, and then applies the gauge-consistent spectral projection.

For orientation, the logical chain developed in this section is

$$\boxed{\begin{array}{l} \mathcal{N}_E = 1 \implies \deg g_- = 1 \implies \deg U_{\rm spin} = 1 \\ \qquad\qquad \xrightarrow{\text{QCD class premise}} B = 1 \implies \text{proton floor} \implies \mathcal{Q}_p = \dfrac{m_p}{M_P}. \end{array}}$$

The only new matter-side physical premise in this chain is the class-level identification $[U_{\rm IR}] = \varphi_*[U_{\rm spin}] = \varphi_*[g_-]$. The primitive integer, the clutching degree, the explicit degree-one spin-holonomy representative, the $B = 1$ consequence, the gauge-consistent proton spectral floor and the normalized Weyl response are derived below once that premise is stated.

5.1 One intrinsic Euler–Chern unit on the round sphere

Define the normalized Euler unit

$$\boxed{\mathcal{N}_E \equiv \frac{1}{64\pi^2}\int_{S^4}\sqrt{g}\,E_4 = \frac{\chi(S^4)}{2} = 1.} \tag{6}$$

For an oriented spin four-manifold, the complex rank-two chiral spin bundles $\Sigma^{\pm}$ obey the standard characteristic-class identities [23, 79]

$$e(TM) = c_2(\Sigma^-) - c_2(\Sigma^+), \qquad p_1(TM) = -2\big(c_2(\Sigma^-) + c_2(\Sigma^+)\big). \tag{7}$$

On S^4, $\int e = \chi = 2$ and $\int p_1 = 3\tau = 0$, so for the chosen orientation

$$\boxed{\langle c_2(\Sigma^-), [S^4]\rangle = +1, \qquad \langle c_2(\Sigma^+), [S^4]\rangle = -1.} \tag{8}$$

Thus the chirality carrying positive Chern number supplies a primitive unit fixed by the oriented round spin geometry rather than by an independent charge selection. In the convention of Eq. (8), Σ^- is the quaternionic Hopf $SU(2)$ bundle over S^4; its Levi–Civita-induced connection is the standard unit instanton [49, 79, 23].

5.2 The equatorial clutching map has degree one

Write

$$S^4 = D^4_N \cup_{S^3} D^4_S.$$

Since each hemisphere is contractible, Σ^- is trivial on each cap and its topology is encoded by the transition function $g_- : S^3 \to SU(2)$. With the standard quaternionic coordinates one may take

$$g_-(x) = x_4\mathbf{1} + i x_j\sigma_j, \qquad x_4^2 + \sum_j x_j^2 = 1. \tag{9}$$

In the fundamental trace convention used below for the QCD baryon current,

$$\boxed{\deg g_- = \frac{1}{24\pi^2}\int_{S^3}\mathrm{Tr}(g_-^{-1}\mathrm{d}g_-)^3 = 1.} \tag{10}$$

The half-sphere curvature integral need not itself be an integer: with a fixed boundary trivialization the integer relative Chern class is represented by the boundary clutching/Chern–Simons transgression [22]. There is therefore no factor-of-two conflict between the two hemispheres and Eq. (8).

Equations (6), (8) and (10) give the exact intrinsic identity

$$\boxed{\mathcal{N}_E = \langle c_2(\Sigma^-), [S^4]\rangle = \deg g_- = 1.} \tag{11}$$

An explicit degree-one spin-holonomy representative. The same class has a connection-level representative that requires no new integer choice. The Levi–Civita-induced connection on Σ^- is the charge-one $SU(2)$ instanton already identified above. Its Atiyah–Manton holonomy in stereographic coordinates defines a group-valued field $U_{\rm spin} : S^3 \to SU(2)_{\rm spin}$ whose degree equals the instanton number, [83]

$$\boxed{\deg U_{\rm spin} = \langle c_2(\Sigma^-), [S^4]\rangle = 1, \qquad [U_{\rm spin}] = [g_-].} \tag{12}$$

Thus the round spin geometry supplies not only an abstract clutching generator but an explicit degree-one Skyrme-type representative. Related constructions from gravitational spin-connection holonomy provide independent precedent [86]. This statement is purely geometric: it does not identify $SU(2)_{\rm spin}$ with the physical QCD flavour group.

5.3 One class-level QCD boundary premise

At the QCD+QED confinement matching slice, the ordinary two-flavour chiral field $U : S^3 \to SU(2)_{\text{flavour}}$ carries the integer winding

$$B[U] = \frac{1}{24\pi^2} \int_{S^3} \mathrm{Tr}(U^{-1}\mathrm{d}U)^3 \in \mathbb{Z}, \tag{13}$$

which is the low-energy baryon number; the Skyrme/QCD identification and its anomaly-matching role are standard [80, 81, 87, 88]. The paper makes the following single matter-side physical identification:

> **QCD state-preparation premise.** At confinement, the infrared QCD chiral boundary state is prepared in the homotopy class of the intrinsic degree-one spin field,
>
> $$\boxed{[U_{\mathrm{IR}}] = \varphi_*[U_{\mathrm{spin}}] = \varphi_*[g_-] \in \pi_3(SU(2)_{\text{flavour}}),}$$
>
> where φ is orientation preserving at the level of the two abstract $SU(2)$ groups.

This class-level identification is the paper's single new matter-side premise. It fixes the integer, its sign and its primitive normalization without any further selection. Equations (10)–(13) give

$$\boxed{B = +1, \qquad B = -1 \text{ for the conjugate orientation.}} \tag{14}$$

Orientation-preserving automorphisms of $SU(2)$ act by conjugation on the primitive class and do not introduce a continuous matching coefficient.

The holonomy construction above makes the Atiyah–Manton connection explicit inside the present geometry rather than using it only as an analogy. Holographic QCD likewise realizes baryons as flavour-gauge instantons, a distinct mechanism, while other constructions relate gravitational instanton topology or spin-connection holonomy to Skyrme/baryonic structure [84, 85, 86]. None of these results forces the physical QCD flavour field to equal U_{spin}; that remaining class-level state-preparation identification is precisely the premise stated above.

Class-level scope of the matching. The matching is strictly between homotopy classes. No connection-level spin–flavour identification is imposed. Microscopic implementations that introduce additional background or interface structure are physically distinct models and must include their associated finite terms in the action budget.

From the $B = 1$ sector to the proton channel. The topological step ends with the $B = 1$ sector; the physical mass is then determined by the QCD+QED Hamiltonian. The proton enters through the long-time spectral projection of the selected $B = 1$ QCD+QED sector, whose lowest state is the proton. QCD generates the underlying infrared scale by dimensional transmutation [47, 48], realized physically in the confined hadronic spectrum. Standard collective-coordinate quantization of the $B = 1$ Skyrmion contains the spin/isospin-1/2 nucleon sector [82]. The long-time projection below therefore selects the proton whenever the interpolating state has the standard nonzero proton overlap.

5.4 Gauge-consistent proton floor

At the QCD+QED matching scale, minimization over gauge-consistent electromagnetic superselection sectors within $B = 1$ gives the proton as the spectral floor; the conjugate $B = -1$ sector gives the antiproton [4]. Thus the proton charge is not an additional state-selection premise after $B = 1$: it is the electromagnetic label of the lowest physical QCD+QED state. Embedding this infrared state into the renormalizable Standard Model does not destabilize the proton; the exact nonperturbative symmetry structure forbids proton decay [72].

A globally closed finite-volume QED system cannot support nonzero total charge. In a factorisation region $\mathcal{R}$, however, gauge-invariant subsystems have superselection sectors labelled by normal electric flux through $\partial\mathcal{R}$ [73, 74, 75, 76]. The minimizing proton sector therefore carries

$$\int_{\partial\mathcal{R}} *F = e, \tag{15}$$

while the conjugate region carries $-e$. Gluing the two caps restores a globally neutral compact state. This electromagnetic edge label is the Gauss-law dressing of the spectral minimum; it does not alter the clutching/baryon winding.

Let L_{IR} denote the size of a charged-state regulator and $E_{B=1,\mathcal{E}}(L_{\mathrm{IR}})$ the lowest energy in the compatible flux sector. This infrared regulator is not the geometric radius of either compact hemisphere: the clutching class labels the prepared infrared topological sector, while L_{IR} regulates the asymptotic QCD+QED charged state. The physical infrared floor is

$$\lim_{L_{\mathrm{IR}}\to\infty} \min_{\mathcal{E}\ \text{compatible}} \left[E_{B=1,\mathcal{E}}(L_{\mathrm{IR}}) - E_0(L_{\mathrm{IR}})\right] = m_p. \tag{16}$$

Finite-volume electromagnetic self-energies and boundary dressings vanish or become subextensive in this ordered limit [73, 76].

5.5 Normalized Weyl slope and Newton-unit covariance

Define the sign of the trace source by the Hamiltonian deformation

$$H(\sigma) = H(0) + \sigma\Theta + O(\sigma^2), \qquad \Theta \equiv \int_\Sigma \sqrt{h}\, T^\mu{}_\mu\, d^3x, \tag{17}$$

so that $H'(0) = \Theta$. This fixes the source convention independently of the geometric Weyl parameter used in Sec. 3. Let $C_{p,\mathcal{E}}(T, L_{\mathrm{IR}}; \sigma)$ be a gauge-invariantly dressed Euclidean correlator in the regulated $B = 1$ sector with this constant trace source. Ground-state projection at fixed regulator gives

$$C_{p,\mathcal{E}}(T, L_{\mathrm{IR}}; \sigma) = Z_p(\sigma) e^{-E_p(\sigma)T}\left[1 + O(e^{-\Delta T})\right]. \tag{18}$$

Theorem 2 (Proton Weyl-slope theorem)**.** *With the trace-source normalization of Eq.* (17)*,*

$$\boxed{-\lim_{T\to\infty} \frac{1}{T}\, \partial_\sigma \ln \frac{C_{p,\mathcal{E}}(T, L_{\mathrm{IR}}; \sigma)}{C_{p,\mathcal{E}}(T, L_{\mathrm{IR}}; 0)}\bigg|_{\sigma=0} = m_p.} \tag{19}$$

Equivalently, for the observable $q = G\Lambda_\infty$ and $M_P = G^{-1/2}$,

$$\boxed{-\lim_{s\to\infty} \frac{1}{s}\, \partial_\sigma \ln \frac{C_p(s;\sigma)}{C_p(s;0)}\bigg|_{\sigma=0} = m_p\sqrt{G} = \frac{m_p}{M_P}, \qquad s = M_P T.} \tag{20}$$

Proof. Equation (18) gives $-T^{-1}\partial_\sigma \ln C_p \to \partial_\sigma E_p$. The field-theoretic Feynman–Hellmann theorem identifies this derivative with the ground-state matrix element of the integrated trace [70]. At rest,

$$\partial_\sigma E_p\big|_{\sigma=0} = \frac{1}{2m_p}\langle p|T^\mu{}_\mu(0)|p\rangle.$$

The exact forward energy–momentum-tensor normalization gives $\langle p|T_{\mu\nu}(0)|p\rangle = 2P_\mu P_\nu$ and hence $\langle p|T^\mu{}_\mu(0)|p\rangle = 2m_p^2$ [71]. Substitution gives Eq. (19); multiplication by $\sqrt{G}$ gives Eq. (20). Multiplicative overlaps, finite edge dressings and other endpoint factors have logarithms $o(T)$ and therefore drop out of the normalized long-time slope. □

Recent lattice calculations directly resolve hadronic trace-anomaly contributions and nucleon trace-anomaly form factors, providing modern nonperturbative checks on the energy–momentum-tensor sector entering this normalization [77, 78].

Reduced-Planck notation is algebraically equivalent when all normalizations are transformed consistently. If $\bar{M}_P = (8\pi G)^{-1/2}$, then

$$\frac{m_p}{M_P} = \frac{1}{\sqrt{8\pi}}\frac{m_p}{\bar{M}_P}, \qquad G\Lambda_\infty = \frac{1}{8\pi}\frac{\Lambda_\infty}{\bar{M}_P^2}. \tag{21}$$

Changing notation consistently transforms both the matter response and the curvature observable. Replacing only one denominator while retaining the coefficient-one source law would instead define a different normalization.

5.6 Normalized matter/IR composite

Let

$$\mathcal{A}_E \equiv a_{\rm eff}, \qquad \mathcal{Q}_p \equiv \frac{m_p}{M_P}$$

be the two separately normalized response coordinates. Define the normalized matter/IR composite by

$$\boxed{q^{(1)}_{\rm matter} \equiv \mathcal{A}_E\mathcal{Q}_p = a_{\rm eff}\frac{m_p}{M_{\rm P}}.} \tag{22}$$

Equation (22) defines this composite with coefficient one; its Euler and proton factors remain separately normalized readouts. The QCD state-preparation premise supplies the physical $B = 1$ sector and the intrinsic geometry fixes its primitive integer normalization. Any coefficient coupling this composite to curvature belongs to the global source pairing stated separately in Sec. 9.

At leading Standard Model order, $a_{\rm eff} \to a_{\rm SM} = 1991/720$. The ratio itself is

$$\frac{m_p}{M_{\rm P}} = 7.685 \times 10^{-20}, \tag{23}$$

using the physical proton mass and $M_P = G^{-1/2}$ [4]. The proton is the spectral floor of the degree-one QCD+QED class prepared by the QCD state-preparation premise. The Euler coefficient and proton scale are dynamically distinct readouts tied to the same primitive geometric unit.

6 The compact de Sitter saddle weight $e^{-24\pi^2}$

The compact extraction theorem identifies the protected matter coordinate; the gravitational sector supplies a distinct semiclassical saddle factor. This section states the strict compact channel used in the numerical result and separates its geometric consequences from the physical choices that define the channel.

Round-sphere action. For the full round Euclidean de Sitter four-sphere [39], $R = 4\Lambda_{\rm UV}$ and $\mathrm{Vol}(S^4) = 24\pi^2/\Lambda_{\rm UV}^2$. Hence

$$S_{\rm EH} = -\frac{1}{2\kappa^2}\int_{S^4}(R - 2\Lambda_{\rm UV})\sqrt{g}\,d^4x = -\frac{24\pi^2}{\kappa^2\Lambda_{\rm UV}} \;=\; -\frac{3\pi}{G\Lambda_{\rm UV}}. \tag{24}$$

A hemisphere instead has action $-3\pi/(2G\Lambda_{\rm UV})$; its equatorial boundary is totally geodesic, so the Gibbons–Hawking boundary term vanishes. The compact weight here uses the full sphere.

Writing $\bar{M}_P = \kappa^{-1}$ and $\rho_{\rm UV} \equiv \Lambda_{\rm UV}/\kappa^2 = \Lambda_{\rm UV}\bar{M}_P^2$, define

$$u \equiv \kappa^2\Lambda_{\rm UV} = \frac{\Lambda_{\rm UV}}{\bar{M}_P^2} = \frac{\rho_{\rm UV}}{\bar{M}_P^4}, \qquad |S_{\rm EH}| = \frac{24\pi^2}{u}.$$

Thus u is the compact saddle vacuum-energy density measured in reduced-Planck-density units. Using the intrinsic unit of Eq. (6), the same round-sphere identity can be written

$$\boxed{|S_{\rm EH}| = \frac{24\pi^2}{u}\,\mathcal{N}_E.} \tag{25}$$

Thus the Euler-anomaly readout, the degree-one clutching class and the classical gravitational action all use the same normalized geometric unit $\mathcal{N}_E = 1$; their dynamics remain distinct.

Independent saddle support. A recent quantum-EFT analysis of the four-dimensional gravitational path integral with $\Lambda > 0$ finds that, to leading order, $\mathcal{Z}_{\rm grav}$ is dominated by the four-sphere saddle and its small fluctuations; additional positive-curvature Einstein geometries enter beyond leading order, with $\mathbb{CP}^2$ treated explicitly as the next-largest-volume example [56]. This supplies independent path-integral support for taking round S^4 as the leading compact positive-curvature gravitational saddle in the present channel.

Canonical topological cross-check. Recent nonperturbative canonical work on de Sitter gravity provides an independent realization of the same underlying architecture. The exact Chern–Simons–Kodama solution of the Wheeler–DeWitt constraint places the quantum-gravitational state in a topological θ sector with $\theta = 12\pi^2/(\Lambda\ell_{\rm Pl}^2) \pmod{2\pi}$, where a unit large $SU(2)$ transformation is normalized by the standard integer winding of $S^3 \to SU(2)$ [60]. Thus primitive $SU(2)$ topology accompanied by an inverse dimensionless cosmological coupling appears independently in exact canonical gravity, closely paralleling the clutching and inverse-u structure above. The round-S^4 calculation here fixes its own coefficient $24\pi^2/u$ and the strict channel retains the boundary prescription stated below.

The strict channel declares the reduced-Planck-density domain

$$0 < \rho_{\rm UV} \le \bar{M}_P^4, \qquad \text{equivalently} \qquad 0 < u \le 1.$$

The scale $\rho_{\rm UV} \sim \bar{M}_P^4$ is the conventional Planck-scale EFT benchmark for vacuum energy [6, 8]. The equality at one should therefore be read as the exact endpoint of the *declared* density domain, not as a theorem that every definition of Planckian curvature or every ultraviolet completion places its breakdown at the same numerical coefficient. In particular,

$$\boxed{u_{\max} = 1 \quad\Longleftrightarrow\quad \rho_{\rm UV} = \bar{M}_P^4.}$$

Within this domain $B(u) = 24\pi^2/u$ increases monotonically as u decreases, so the least-suppressed member is the upper endpoint. This monotonicity orders the members of the declared family but does not itself define an integration measure over u. The strict compact observable used here adopts the least-suppressed boundary member as the member evaluated; no modulus integration over u is part of this defined compact channel. This boundary-member prescription is a selected compact-sector prescription, distinct from the separate selection of the exact boundary normalization $u_{\max} = 1$. Thus

$$\boxed{u_b = u_{\max} = 1.} \tag{26}$$

At that boundary,

$$\boxed{B_1 \equiv |S_{\rm EH}|_{u_b=1} = 24\pi^2.} \tag{27}$$

The coefficient $24\pi^2$ is the dimensionless Einstein–Hilbert action at the declared reduced-Planck vacuum-density boundary. This compact S^4 is an ultraviolet gravitational sector of the construction: $u_b = 1$ fixes $\Lambda_{\rm UV} = \bar{M}_P^2$ and therefore its input weight. The late-time curvature Λ_∞ is a separate output of the proposed global-source map in Sec. 9; $\Lambda_{\rm obs}$ is used only for comparison. The saddle action is evaluated at $\Lambda_{\rm UV}$, not at Λ_∞ or $\Lambda_{\rm obs}$. The endpoint is fixed without reference to the observed cosmological constant, but the choice of $\rho_{\rm UV} \le \bar{M}_P^4$ as the ultraviolet domain of this compact channel remains a physical selection to be justified by a microscopic gravity calculation. For comparison, Padmanabhan's independent mode-count construction gives $\Lambda L_P^2 = 3e^{-24\pi^2\mu}$ [89]; its exponent arises from cosmic mode counting rather than a round-S^4 saddle.

The magnitude of the on-shell action equals the de Sitter entropy [39],

$$B = S_{\rm dS} = \frac{3\pi M_{\rm P}^2}{\Lambda_{\rm UV}}, \tag{28}$$

and the Chern–Gauss–Bonnet theorem gives on a conformally flat Einstein four-manifold

$$|S_{\rm EH}| = \frac{3}{2}\,\chi(M)\,\frac{8\pi^2}{\kappa^2\Lambda_{\rm UV}}. \tag{29}$$

For S^4, $\chi = 2$, recovering Eq. (24) [21, 24]. In four dimensions the relevant topological density is precisely E_4, the same Euler four-form that appears in the type-A trace anomaly. These are geometric identities; they do not select a contour.

Suppressed-thimble clause. The cosmological channel considered here is defined so that the compact round-S^4 thimble is included with the decaying orientation $e^{-|S_{\rm EH}|}$ [39, 40, 66]. The decaying thimble is the contour choice that defines this compact observable. The broader no-boundary contour problem remains sensitive to Stokes data, boundary conditions and the chosen observable [64, 41, 42, 54, 55]. Recent analytically continued Chern–Simons work constructs explicitly the Lefschetz thimble attached to the self-dual de Sitter saddle and finds Gaussian damping of anisotropic directions about the symmetric configuration, extending the de Sitter thimble picture beyond the isotropic reduction [61]. This supplies a concrete independent realization of the semiclassical thimble architecture used here. The decaying sign coincides semiclassically with the tunnelling/Vilenkin sign [65]; the opposite sign would replace the suppression by an enormous enhancement and is not the channel analysed here.

Classical strict-channel factor. For the primitive boundary member,

$$\boxed{e^{-B_1} = e^{-24\pi^2} = 1.344 \times 10^{-103}.} \tag{30}$$

This is the classical Einstein–Hilbert saddle factor of the compact ultraviolet source sector, distinct from a thermal partition function for the observed late-time universe.

Determinant and higher-curvature status. No universal absolute one-loop constant is assigned to the sphere amplitude. On round S^4, finite R^2 and Euler counterterms evaluate to arbitrary constants, so the finite constant part of a raw determinant is not a scheme-independent observable without an additional renormalization condition. Recent one-loop work on de Sitter no-boundary amplitudes likewise keeps nontrivial determinant, measure and symmetry-volume factors explicit [62]. The strict leading formula therefore retains the classical saddle factor Eq. (30) and treats the complete source-dependent gravitational dressing and higher-curvature change of the Planck-boundary action as open corrections. Appendix A gives the short determinant audit. Because $B_1 \simeq 237$, even an order-one shift in the complete primitive action would materially change the result; this is a central ultraviolet test of the channel.

7 Leading compact-channel result

The leading compact response combines the normalized matter/IR composite with the classical saddle factor. Section 5 defines $q^{(1)}_{\text{matter}} = a_{\text{eff}}(m_p/M_{\text{P}})$ from two separately normalized response coordinates. The intrinsic spin geometry fixes their common primitive geometric unit, while the QCD state-preparation premise fixes the physical $B = 1$ QCD realization. The strict compact gravitational channel of Sec. 6 supplies $e^{-24\pi^2}$. Define the strict leading compact value by

$$\boxed{q_{\text{comp}} \equiv q^{(1)}_{\text{matter}} e^{-24\pi^2} = a_{\text{eff}} \frac{m_p}{M_{\text{P}}} e^{-24\pi^2}.} \tag{31}$$

The assembly has two defined stages: the separately normalized matter readouts first form q_{matter}, after which the specified compact saddle supplies its gravitational factor. The coefficient-one factorization in Eq. (31) is the definition of the strict compact-channel observable assembled from these independently normalized responses; it is not claimed as a separate dynamical theorem.

Leading strict-channel benchmark. With the QCD state-preparation premise and the selected $u_b = 1$ decaying compact gravitational sector, the leading value is

$$\boxed{q_{\text{comp}} = a_{\text{SM}} \frac{m_p}{M_{\text{P}}} e^{-24\pi^2} = 2.856 \times 10^{-122}.} \tag{32}$$

Numerical audit. The complete leading arithmetic is

$$\begin{aligned} a_{\text{SM}} &= \frac{1991}{720} = 2.76527778, \\ \frac{m_p}{M_{\text{P}}} &= 7.68514844 \times 10^{-20}, \\ e^{-24\pi^2} &= 1.34414611 \times 10^{-103}, \\ q_{\text{comp}} &= (2.76527778)(7.68514844 \times 10^{-20})(1.34414611 \times 10^{-103}) \\ &= 2.85652154 \times 10^{-122}. \end{aligned} \tag{33}$$

Thus the hierarchy separates into roughly $103 + 19$ decades: the compact saddle supplies the larger suppression, the QCD-to-Planck ratio supplies the remaining hierarchy, and the Euler coefficient is order unity. Within this channel the hierarchy is multiplicative rather than a cancellation among large vacuum-energy terms.

Table 1: Status of the ingredients and assembly definitions at the point where the strict compact response is assembled. Downstream global-source assumptions are not included because they enter only when $q_{\rm comp}$ is mapped to cosmological curvature.

Ingredient	Role in the compact result	Status
Euler extraction	unique protected local compact datum	derived
$a_{\rm SM} = 1991/720$	leading free-field Standard Model Euler coefficient	computed
Interacting $a_{\rm eff}$	exact compact-channel matter coefficient beyond leading order	open calculation
Intrinsic Euler–Chern unit	$\mathcal{N}_E = \chi/2 = c_2(\Sigma^-) = \deg g_- = 1$	derived
Explicit spin-holonomy representative	$\deg U_{\rm spin} = 1$ and $[U_{\rm spin}] = [g_-]$	derived
QCD state-preparation class identification	$[U_{\rm IR}] = \varphi_*[U_{\rm spin}] = \varphi_*[g_-]$	structural premise
$B = 1 \to m_p/M_P$	degree-one sector, proton spectral floor and normalized Weyl slope	derived given premise
Newton-unit normalization	$M_P = G^{-1/2}$ and the corresponding reduced-Planck conversion	normalization covariance
Factorized compact assembly	$q_{\rm comp} \equiv [a_{\rm eff}(m_p/M_P)]e^{-24\pi^2}$ from separately normalized response coordinates	defined strict-channel composite
Monotonic ordering	the largest allowed u is the least-suppressed member since $e^{-24\pi^2/u}$ increases with u	derived given strict domain
Boundary-member prescription	evaluate the compact channel at the least-suppressed member $u_b = u_{\rm max}$; no modulus integration over u is part of the defined compact observable	selected compact-sector prescription
Exact boundary normalization	$u_{\rm max} = 1 \iff \rho_{\rm UV} = \bar{M}_P^4$, the endpoint of the declared reduced-Planck-density domain	selected UV domain
Decaying thimble	suppressed orientation of the compact S^4 saddle	selected contour
$e^{-24\pi^2}$	classical round-S^4 factor at $u_{\rm max} = 1$	derived given member, boundary and contour selections
Gravitational dressing	source-dependent determinant and higher-curvature shift of the primitive action	open calculation

The global four-form source law is deliberately absent from Table 1: Eq. (32) is a compact response, not yet a cosmological constant. The one-loop determinant and remaining gravitational corrections are quantified in Sec. 10 and Appendix A.

The quantity obtained so far is therefore a compact response, not yet a cosmological curvature. We next ask whether ordinary semiclassical Einstein gravity can convert it into a small curvature linear in $q_{\rm comp}$.

8 Ordinary-GR obstruction and the trace-free quotient

The ordinary-GR obstruction. Let Λ_g denote the geometric cosmological constant of a round four-dimensional de Sitter solution. Then

$$R_{\mu\nu} = \Lambda_g g_{\mu\nu}, \qquad R = 4\Lambda_g, \qquad E_4 = \frac{8}{3}\Lambda_g^2.$$

With the anomaly convention $\langle T^\mu{}_\mu\rangle_A = -aE_4/(16\pi^2)$, maximal symmetry fixes

$$T^A_{\mu\nu} = -\frac{a\Lambda_g^2}{24\pi^2} g_{\mu\nu}.$$

Proposition 3 (Ordinary-GR no-go for a linear compact source)**.** *In ordinary semiclassical Einstein gravity the local type-A anomaly changes the round de Sitter curvature at order $\Lambda_g^2/M_{\rm P}^2$. Multiplying the anomaly sector by a small compact weight does not create a small branch linear in that weight.*

Proof. With a renormalised local cosmological term Λ_0,

$$G_{\mu\nu} + \Lambda_0 g_{\mu\nu} = \frac{8\pi}{M_{\rm P}^2} T^A_{\mu\nu}.$$

Since $G_{\mu\nu} = -\Lambda_g g_{\mu\nu}$ on the round branch,

$$\Lambda_g - \Lambda_0 = \frac{a}{3\pi} \frac{\Lambda_g^2}{M_{\rm P}^2}. \tag{34}$$

For $\Lambda_0 = 0$, the nonzero root is $\Lambda_g/M_{\rm P}^2 = 3\pi/a$, which is Planckian. If the anomaly sector is multiplied by a small coefficient ε, the nonzero root becomes $3\pi/(a\varepsilon)$: the suppression appears in the denominator. Thus no ordinary local metric variation yields $\Lambda_g/M_{\rm P}^2 \propto \varepsilon a$. $\square$

The obstruction is structural. A protected dimensionless datum q could multiply a local volume term $qM_{\rm P}^4 \int \sqrt{|g|}$, but that is precisely the operator removed by the cosmological-counterterm quotient. A two-derivative term $qM_{\rm P}^2 \int \sqrt{|g|}R$ only renormalises Newton's constant, while curvature-squared terms produce qH^4 stresses. A linear protected relation therefore cannot arise from these local metric terms; the completion considered below supplies it through a separate global zero-mode equation.

The local equation on the volume-counterterm quotient. Write the two-derivative renormalised metric equation, before selecting a cosmological representative, as

$$\mathcal{E}_{\mu\nu} \equiv G_{\mu\nu} - \frac{8\pi}{M_{\rm P}^2} T_{\mu\nu}.$$

Adding a finite local volume term shifts $\mathcal{E}_{\mu\nu} \mapsto \mathcal{E}_{\mu\nu} + cg_{\mu\nu}$.

Lemma 4 (Trace-free quotient projector)**.** *The unique algebraic local projection of the metric equation that is independent of the representative $\mathcal{E}_{\mu\nu} \sim \mathcal{E}_{\mu\nu} + cg_{\mu\nu}$ is*

$$R_{\mu\nu} - \frac{1}{4} R g_{\mu\nu} = \frac{8\pi}{M_{\rm P}^2} \left(T_{\mu\nu} - \frac{1}{4} T g_{\mu\nu} \right). \tag{35}$$

For conserved matter stress, any solution of this projected equation leaves one spacetime constant $\Lambda_{\rm int}$ through

$$R + \frac{8\pi}{M_{\rm P}^2} T = 4\Lambda_{\rm int}. \tag{36}$$

Proof. The unique algebraic projection of a symmetric tensor that is invariant under $X_{\mu\nu} \mapsto X_{\mu\nu} + cg_{\mu\nu}$ and acts as the identity on traceless tensors is $X_{\mu\nu} - Xg_{\mu\nu}/4$. Applying it to $\mathcal{E}_{\mu\nu} = 0$ gives Eq. (35). The contracted Bianchi identity and $\nabla^\mu T_{\mu\nu} = 0$ then imply $\nabla_\nu(R + 8\pi T/M_{\rm P}^2) = 0$, giving Eq. (36). $\square$

The resulting trace-free Einstein structure and the appearance of the cosmological constant as an integration constant have a long history and remain active subjects of covariant formulations [28, 29].

Renormalisation ambiguity alone does not select the physical dynamics. Lemma 4 identifies only the local equation that a source law compatible with the volume-counterterm quotient must reproduce. The compact global-source action of Sec. 9 supplies the specific source law used here to fix that omitted scalar mode. Its variation yields both Eq. (35) and the equation fixing the omitted scalar mode.

9 From compact response to cosmological curvature: global-source completion

Ordinary local variation of the type-A anomaly cannot generate a small curvature linear in the compact response (Sec. 8). A separate equation for the scalar zero mode is therefore required. We adopt a Henneaux–Teitelboim/sequestering-type four-form completion for this scalar zero mode [28, 30, 31].

For a connected finite-volume representative define

$$\langle X\rangle = \frac{\int_M \sqrt{|g|}\, X\, d^4x}{\int_M \sqrt{|g|}\, d^4x}.$$

Let $F_4 = dA_3$ be the volume four-form, Λ_b its constraint multiplier, and η a rigid dimensionless Newton variable enforced by a three-form C_η. The branch Newton mass is $M_N^2 \equiv \eta M_{\rm P}^2$. For a fixed rigid dimensionless compact response q we take

$$\boxed{S_{\rm gs}[q] = \int_M \left[\left(\frac{\eta M_{\rm P}^2}{16\pi}R - \mathcal{L}_m\right)\star 1 + \Lambda_b(F_4 - \star 1) - \frac{\eta^2 M_{\rm P}^4}{8\pi} q F_4\right] + \frac{1}{2\pi}\int_M C_\eta \wedge d\eta.} \tag{37}$$

The compact calculation supplies $q = q_{\rm comp}$. The coefficient of q in Eq. (37) is the proposed unit compact-to-global source pairing; it is not fixed by the extraction theorem. This identification is the independent gravity-side completion, and the anomaly extraction theorem itself does not depend on it.

Theorem 5 (Global-source field equations)**.** *The variations of Eq.* (37)*, at fixed rigid q, imply*

$$\langle R\rangle = 4\eta M_{\rm P}^2 q, \tag{38}$$

and

$$\boxed{G_{\mu\nu} + \eta M_{\rm P}^2 q\, g_{\mu\nu} = \frac{8\pi}{\eta M_{\rm P}^2}\left(T_{\mu\nu} - \frac{1}{4}\langle T\rangle g_{\mu\nu}\right).} \tag{39}$$

Consequently homogeneous shifts $\mathcal{L}_m \to \mathcal{L}_m + \rho$ cancel exactly. For a maximally symmetric vacuum on any fixed branch, writing its cosmological curvature as Λ_∞,

$$\boxed{\frac{\Lambda_\infty}{M_N^2} = q, \qquad M_N^2 = \eta M_{\rm P}^2.} \tag{40}$$

Thus η drops out when the result is expressed in the measured branch Newton unit; setting $\eta = 1$ only identifies the reference symbol $M_{\rm P}$ with M_N for the quoted numerical branch. It is not an additional physical selection. The residual source is constant on a fixed branch and has $w = -1$.

Proof. Variation of Λ_b gives $F_4 = \star 1$. Variation of A_3 makes the coefficient conjugate to F_4 spacetime constant. The rigid η variation, with q held fixed, gives $\langle R\rangle = 4\eta M_{\rm P}^2 q$. Metric variation gives

$$\frac{\eta M_{\rm P}^2}{8\pi} G_{\mu\nu} + \Lambda_b g_{\mu\nu} = T_{\mu\nu}.$$

Using the averaged trace together with Eq. (38) eliminates Λ_b and gives Eq. (39). A constant shift of the matter Lagrangian changes $T_{\mu\nu}$ and $\langle T\rangle g_{\mu\nu}/4$ equally and therefore cancels. For a maximally symmetric vacuum Eq. (40) follows after writing the result in terms of $M_N^2 = \eta M_{\rm P}^2$. □

After matching the reference parameter to the measured Newton branch ($M_N = M_{\mathrm{P}}$, equivalently $\eta = 1$ in this parametrization), Eq. (39) can be written

$$G_{\mu\nu} + \Lambda_{\mathrm{res}} g_{\mu\nu} = \frac{8\pi}{M_{\mathrm{P}}^2} T_{\mu\nu}, \qquad \Lambda_{\mathrm{res}} = M_{\mathrm{P}}^2 q + \frac{2\pi}{M_{\mathrm{P}}^2} \langle T \rangle. \tag{41}$$

The residual term is exactly proportional to the metric, so its equation of state is $w = -1$ wherever it is identified with dark energy. The trace average fixes the amplitude, not the equation of state.

On round S^4 the local anomaly stress is homogeneous and therefore drops out of $T_{\mu\nu} - \langle T \rangle g_{\mu\nu}/4$. The local H^4 response of Sec. 8 is removed by the trace-free projection, while nonhomogeneous stresses and ordinary finite-wavelength matter excitations continue to gravitate through Eq. (39). The protected compact response survives separately in the rigid source q.

For a cosmological history, local late-time vacuum does not by itself imply $\langle T \rangle = 0$. Let $\{M_\tau\}$ be a nested future-directed exhaustion of the selected Lorentzian branch with four-volume $V_4(M_\tau) \to \infty$. Define

$$\langle T_{\mathrm{nonvac}} \rangle_{\mathrm{reg}} \equiv \lim_{\tau \to \infty} \frac{\int_{M_\tau} \sqrt{-g}\, T_{\mathrm{nonvac}}\, d^4x}{\int_{M_\tau} \sqrt{-g}\, d^4x}. \tag{42}$$

The admitted regulator class consists of connected exhaustions of the same future branch for which boundary/edge contributions are subextensive in V_4 and the ratio above has the same limit. Direct comparison with a constant asymptotic cosmological term then assumes

$$\boxed{\langle T_{\mathrm{nonvac}} \rangle_{\mathrm{reg}} = 0.} \tag{43}$$

For the late-time identification, this limit must be regulator-independent within the admitted exhaustion class. On a future-eternal asymptotically de Sitter branch, ordinary diluted matter of finite comoving abundance gives a subextensive integrated trace, while radiation is trace-free, so the condition is satisfied in that standard late-time regime. Finite-duration phase-transition epochs are likewise subextensive under the same future exhaustion and do not change the asymptotic average. Under this condition,

$$\frac{\Lambda_\infty}{M_{\mathrm{P}}^2} = q_{\mathrm{comp}}. \tag{44}$$

Combining with Eq. (32) gives

$$\boxed{\frac{\Lambda_\infty}{M_{\mathrm{P}}^2} = a_{\mathrm{SM}} \frac{m_p}{M_{\mathrm{P}}} e^{-24\pi^2} = 2.856 \times 10^{-122}.} \tag{45}$$

The numerical factors have already been audited at the compact-assembly stage in Eq. (33). The Planck 2018 flat-ΛCDM-inferred value is $(2.846 \pm 0.057) \times 10^{-122}$, a 0.4% central-value difference [3]. This is a comparison with that cosmological-model inference, not a direct measurement of an asymptotic constant. No continuous parameter in the compact formula is adjusted to make the comparison.

10 Correction budget, predictions, and tests

The compact-channel status is exposed at the point of assembly in Table 1. The observational comparison in Eq. (45) now gives a quantitative tolerance for the remaining open corrections and downstream completion conditions.

Interacting matter coefficient. The rational value $a_{\rm SM} = 1991/720$ is the leading free-field Standard Model Euler coefficient. The full interacting curved-space Standard Model value $a_{\rm eff}$ has not been calculated in the precise compact channel used here. Existing local-RG and interacting-field results constrain its structure [26, 12, 50] but do not justify importing a numerical uncertainty estimate. A complete calculation would directly test the leading approximation $a_{\rm eff} \to a_{\rm SM}$.

Intrinsic unit and QCD state-preparation premise. The round spin geometry fixes the primitive integer rather than selecting it from an independent charge lattice: Eqs. (6)–(11) give $\mathcal{N}_E = c_2(\Sigma^-) = \deg g_- = 1$. The single remaining matter-side structural premise is that the infrared QCD chiral state is prepared in the homotopy class of that clutching map. Given this premise, $B = 1$ and the proton spectral floor follow without a separate integer-map or projector normalization. A microscopic derivation of the spin-to-flavour class relation from Standard Model plus Einstein dynamics remains open. The construction matches homotopy classes only and therefore does not inherit a connection-level spin–flavour twist. Planck-mass notation introduces no additional physical choice: Eq. (21) is the algebraic conversion between $q = G\Lambda_\infty$ and reduced-Planck variables. The remaining matter questions are the microscopic origin of the class-level boundary condition and the interacting value $a_{\rm eff}$.

Strict compact gravitational channel. The boundary-member prescription, exact reduced-Planck-density normalization $u_{\max} = 1$, and decaying compact thimble together define the strict $u_b = 1$ gravitational channel. The classical saddle factor is exactly $e^{-24\pi^2}$ within the Einstein–Hilbert saddle calculation. The absolute finite sphere determinant is not a canonical universal number (Appendix A). Source-dependent gravitational dressing and higher-curvature contributions to the complete primitive action remain open. The leading benchmark is the Einstein–Hilbert truncation $\delta B = 0$; this is not a statement that the raw finite determinant prefactor $C_{\rm grav}$ has a canonical value of one. Write the net physical exponent shift from the open gravitational corrections as δB. Holding the matter coefficient at its leading value $a_{\rm eff} = a_{\rm SM}$, the gravity-only slice is $q(\delta B) = q_0 e^{-\delta B}$ with $q_0 = 2.8565 \times 10^{-122}$. Relative to the quoted Planck 2018 inference [3], remaining inside its 1σ interval on this slice requires

$$\boxed{-0.0161 \lesssim \delta B \lesssim 0.0239,} \tag{46}$$

and the 2σ interval requires

$$\boxed{-0.0356 \lesssim \delta B \lesssim 0.0446.} \tag{47}$$

Thus, at fixed $a_{\rm eff} = a_{\rm SM}$, the relevant ultraviolet tolerance is at the few-times-10^{-2} level in the action, not merely "smaller than order one." More generally the benchmark constrains the combination $\delta B - \ln(a_{\rm eff}/a_{\rm SM})$; the intervals above are the gravity-only slice of that joint matter–gravity correction space. Controlling these terms is the most important quantitative gravity-side calculation.

Ultraviolet implementation dependence. Any microscopic implementation that introduces additional finite interface or background-field terms must include them in δB (or in $a_{\rm eff}$ when they belong to the projected anomaly coordinate). The bounds in Eqs. (46)–(47) therefore give a direct selection criterion on the gravity-only slice: implementations whose finite cap terms exceed that budget fail the leading benchmark unless another open sector supplies a compensating correction.

Completion-specific status. The field equations in Theorem 5 follow from the stated four-form action, but selection of that source law by the microscopic theory, including unit coupling to the normalized compact response, remains proposed. The quoted late-time amplitude also requires the regulator-independent exhaustion and vanishing non-vacuum trace average in Eqs. (42)–(43). These inputs are downstream of the compact result summarized in Table 1; failure of either completion condition would invalidate the cosmological identification without undoing the compact extraction or the matter/IR construction.

Observational tests. The leading strict-channel value differs from the Planck 2018 flat-ΛCDM central inference by 0.4% (0.18σ) [3]. That number is a benchmark, not a theory uncertainty. The global-source completion predicts a spacetime-constant residual with $w = -1$. DESI DR2 combinations with CMB and supernova data, and the full Dark Energy Survey multi-probe analysis, have reported dataset-dependent preferences for evolving-dark-energy regions [43, 44]. Persistent, cross-dataset and systematics-robust evidence for evolution away from a constant source would falsify the proposed global-source identification while leaving the compact extraction and matter/IR bridge as separate statements.

The strict gravitational boundary is also sharply testable: a change in the complete primitive action changes the prediction exponentially, with the allowed benchmark range quantified in Eqs. (46)–(47). Conversely, changes in microscopic field content affect the leading result through the protected matter coordinate a_{eff} and, if they alter the microscopic realization of the class-level spin-to-QCD boundary condition, through any newly introduced finite cap terms. Without those calculations the field-content sensitivity remains qualitative.

11 Discussion

The results separate into a compact extraction theorem and a conditional cosmological construction. The extraction theorem shows that, after the volume-counterterm ambiguity is removed, the local linear constant-Weyl response on conformally flat S^4 contains one protected matter coordinate, the Euler/type-A coefficient. The conceptual consequence is that, within this compact local response, the scheme-independent matter question is not the value of an absolute vacuum energy but the Euler coordinate that survives the counterterm quotient. This theorem and the leading Standard Model value $a_{\text{SM}} = 1991/720$ do not depend on the later QCD, contour or global-source construction.

The cosmological construction asks whether the same compact saddle can support a numerical cosmological channel. Here the identity

$$\mathcal{N}_E = \frac{\chi(S^4)}{2} = \langle c_2(\Sigma^-), [S^4] \rangle = \deg g_- = \deg U_{\text{spin}} = 1$$

provides a common primitive geometric unit for three dynamically distinct readouts: the Euler anomaly coefficient, the degree-one infrared QCD sector, and the round Einstein action. The matter bridge is a class-level boundary condition rather than a connection-level spin–flavour lock, and the proton is the spectral floor of the QCD+QED sector. The only new matter-side physical premise is that the QCD chiral state is prepared in the corresponding clutching homotopy class.

Independent structural convergence

The complete channel assembled here is not inherited from any existing cosmological-constant proposal. Several of its structural ingredients, however, have been selected independently in approaches motivated by the same radiative-stability problem. This convergence is useful as a prior on the architecture, not as validation of the numerical result.

First, formulations in which the cosmological constant is an integration or global variable provide a close precedent for separating homogeneous vacuum shifts from the scalar-curvature mode. Henneaux–Teitelboim gravity and local vacuum-energy sequestering implement that separation with nonpropagating four-form sectors [28, 30, 31]. In the extension of sequestering that includes graviton loops, the four-dimensional Gauss–Bonnet invariant is singled out as a minimal topological curvature variable: its integral changes only the topological sector and its topological character protects the corresponding rigid coupling from additional graviton-loop dependence [32]. That mechanism is different from the compact response used here, but it independently joins two ingredients that also appear in the present construction: an Euler/Gauss–Bonnet sector and a nonpropagating four-form global sector.

Second, anomaly effective-action approaches provide a more direct structural parallel between those two ends of the channel. In Mottola's effective theory of dynamical vacuum energy, the Euler–Gauss–Bonnet term in the conformal anomaly is related to its Chern–Simons three-form, and a torsion-dependent part of that three-form is identified with an Abelian three-form gauge potential whose four-form field strength describes dynamical vacuum energy [51]. The present construction does not use that torsional identification and its global-source action is different, but the independent chain

$$\text{conformal anomaly} \longrightarrow \text{Euler/Chern–Simons structure} \longrightarrow \text{four-form vacuum sector}$$

shows that the anomaly–Euler–four-form combination is not peculiar to the compact formula developed here.

Third, the gravitational and infrared parts each have separate precedents. Euclidean de Sitter and tunnelling/no-boundary constructions have long assigned semiclassical exponential weights to compact de Sitter saddles [39, 64, 65, 41, 54]. A recent quantum-EFT treatment goes further by finding the $\Lambda > 0$ gravitational partition function leading-order dominated by the four-sphere saddle and its small fluctuations [56]. More directly, nonperturbative canonical de Sitter gravity now exhibits a primitive large-$SU(2)$ topological sector weighted by an inverse dimensionless cosmological coupling [60], while analytically continued Chern–Simons gravity supplies an explicit Lefschetz thimble through the self-dual de Sitter saddle with damped anisotropic directions [61]. These results independently reinforce the compact-saddle, primitive-topology and de Sitter-thimble architecture of the gravity sector. Padmanabhan's mode-count construction supplies a further, dynamically different occurrence of the coefficient $24\pi^2$ [89].

At the ultraviolet structural level, recent pregeometric gauge constructions provide concrete examples in which the Einstein–Hilbert term, cosmological constant and four-dimensional topological invariants—including Gauss–Bonnet—descend together from a common symmetry-broken parent [57]. A related realization makes the cosmological constant discrete through a periodic gravitational topological angle and ties the sector label to de Sitter entropy [58]. Although quantitatively distinct from the present channel, these constructions provide explicit model realizations of a UV linkage between dynamical gravity, Euler/Gauss–Bonnet topology and the cosmological sector.

The boundary-member prescription, exact reduced-Planck-density normalization $u_{\max} = 1$, and decaying compact contour remain the three channel data specified in Sec. 6. On the matter side, proposals based on the QCD trace anomaly and on the low-energy chiral QCD sector in nontrivial spacetime provide independent precedent for looking to QCD rather than to an arbitrary inserted infrared scale [52, 53]. Recent Standard Model work in Eguchi–Hanson gravitational-instanton backgrounds also shows that quantized $\mathbb{Z}_6^{(1)}$ flux sectors impose global consistency conditions on quark and lepton wavefunctions and induce baryon- and lepton-number violating processes [59]. This supplies a direct precedent for gravitational topology acting on genuine Standard Model global-charge structure, through a mechanism distinct from the class-level $B = 1$ boundary condition used here.

Taken together, these comparisons do not establish the complete channel or raise any of its proposed clauses to theorem status. They show instead that several ingredients of the construction—vacuum-shift quotienting, protected Euler/topological data, nonpropagating four-form variables, compact de Sitter saddle weighting, and a possible QCD infrared role—recur independently in the cosmological-constant literature. What remains distinctive and load-bearing here is correspondingly narrower: the class-level QCD state-preparation condition $[U_{\rm IR}] = \varphi_*[U_{\rm spin}] = \varphi_*[g_-]$, the selected boundary-member prescription together with the exact $u_{\max} = 1$ UV-domain normalization and decaying compact contour, and the unit compact-to-global source pairing. These are therefore the appropriate targets for a microscopic derivation rather than assumptions to be inferred from the numerical agreement.

The remaining program is concentrated in five tests: compute the interacting curved-space value $a_{\rm eff}$; derive the QCD state-preparation class relation microscopically; derive from microscopic gravity the boundary-member prescription, exact $u_{\max} = 1$ UV-domain normalization and decaying contour that together define the strict $u_b = 1$ sector; control source-dependent quantum and higher-curvature corrections within the δB budget of Sec. 10; and derive the four-form source selection together with the regulator-independent late-time trace-average condition. The numerical agreement provides a sharp benchmark for these tests.

12 Conclusion

On compact conformally flat S^4, an absolute homogeneous vacuum-energy term is not a separately protected matter observable because it is degenerate with the cosmological counterterm. The counterterm quotient instead leaves one nonvanishing local linear constant-Weyl datum: the type-A Euler anomaly coefficient. For the minimal Standard Model its leading free-field value is

$$\boxed{a_{\rm SM} = \frac{1991}{720}.}$$

This theorem is independent of the subsequent QCD, contour and global-source construction.

The subsequent compact-channel construction uses the round sphere's intrinsic identity

$$\frac{\chi(S^4)}{2} = \langle c_2(\Sigma^-), [S^4] \rangle = \deg g_- = \deg U_{\rm spin} = 1.$$

With one explicit class-level premise that the infrared QCD state is prepared in the corresponding degree-one homotopy class, the physical QCD+QED transfer problem gives the proton response m_p/M_P. The same normalized Euler unit writes the round gravitational action as $24\pi^2/u$. The boundary-member prescription, exact $u_{\max} = 1$ UV-domain normalization and decaying contour together define the strict $u_b = 1$ sector; there the leading compact-channel value is therefore

$$\boxed{q_{\rm comp} = a_{\rm SM} \frac{m_p}{M_P} e^{-24\pi^2} = 2.856 \times 10^{-122}.}$$

Its central value differs from the Planck 2018 flat-ΛCDM inference by 0.4% [3], with no continuous parameter fitted to that value.

Ordinary local metric variation of the anomaly cannot generate a small branch linear in this compact response. A proposed four-form completion supplies the missing scalar-curvature source equation, cancels homogeneous vacuum shifts and gives a fixed-branch residual with $w = -1$. Direct identification of the asymptotic amplitude additionally requires the stated regulated trace-average condition.

The next decisive tests are to compute the full interacting $a_{\rm eff}$, derive the QCD state-preparation class relation microscopically, derive from microscopic gravity the boundary-member prescription, exact $u_{\max} = 1$ UV-domain normalization and decaying contour that together define the strict $u_b = 1$ sector, control quantum and higher-curvature corrections to the compact action at the required δB precision, and derive the global source law together with the regulator-independent late-time trace-average condition. These tests determine whether the compact channel becomes a full dynamical prediction; the extraction theorem already stands independently of them.

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Data Availability

No original data were generated. Numerical inputs are taken from the published sources cited in the text.

A Determinant status of the strict compact weight

A conventional one-saddle expression is [40, 62]

$$Z_{S^4} = C_{\rm grav} e^{-B_{\rm cl}}, \tag{48}$$

where $C_{\rm grav}$ includes gauge-fixed determinants, zero-mode measures, collective-coordinate factors and contour data. The strict leading result of Sec. 6 uses the classical saddle factor $e^{-B_{\rm cl}}$; it does not assume that the absolute finite coefficient $C_{\rm grav}$ is universally one.

The renormalized gravitational effective action is defined up to finite local terms [36, 14]

$$\Gamma[g] \mapsto \Gamma[g] + \int d^4x\sqrt{g}\,\left(\alpha + \beta R + \gamma R^2 + \delta E_4 + \zeta W^2 + \xi \Box R\right). \tag{49}$$

On a round S^4,

$$\int \sqrt{g}\, R^2 = 384\pi^2, \qquad \int \sqrt{g}\, E_4 = 64\pi^2, \qquad W^2 = 0, \qquad \Box R = 0.$$

Finite shifts of γ or δ therefore change the absolute sphere effective action by arbitrary constants without changing the local closed- sphere equations.

Theorem 6 (No canonical absolute determinant constant)**.** *Within ordinary semiclassical gravity on round S^4, the finite constant part of a gauge-fixed one-loop determinant is not a scheme-independent observable. In particular, a raw statement $C_{\rm grav} = 1$ requires an extra renormalization and measure prescription and is not used in the compact formula.*

Proof. The finite determinant contains constants that can be shifted by the allowed finite R^2 and Euler terms displayed above. Without an external renormalization condition, those pieces cannot be separated into an invariant absolute determinant normalization. $\square$

This does not remove quantum-gravitational corrections. A source-dependent determinant, a higher-curvature contribution that changes the complete Planck-boundary action, a Stokes transition or a nonperturbative change of the selected compact sector can alter the strict leading result. Such effects therefore belong to the correction budget rather than to an arbitrary absolute determinant normalization.

B Off-shell variation of the global-source action

For completeness, the action of Eq. (37) is

$$S_{\rm gs}[q] = \int_M \left[\left(\frac{\eta M_{\rm P}^2}{16\pi} R - \mathcal{L}_m \right) \star 1 + \Lambda_b (F_4 - \star 1) - \frac{\eta^2 M_{\rm P}^4}{8\pi} q F_4 \right] + \frac{1}{2\pi} \int_M C_\eta \wedge d\eta.$$

The independent variables are varied before matching the reference parameter to the measured Newton branch. Writing $M_N^2 = \eta M_{\rm P}^2$ makes clear that η is a parametrization of the branch Newton unit rather than an extra prediction input. The four-form F_4 is metric independent off shell; the relation $F_4 = \star 1$ follows from the Λ_b equation only after variation.

The η^2 dependence of the source term is fixed, up to an overall pairing coefficient, by covariance under this branch-unit parametrization. Replace it temporarily by a general function $f(\eta)$,

$$-\frac{M_{\rm P}^4}{8\pi} f(\eta) q F_4.$$

The rigid variation then gives

$$\langle R \rangle = 2 M_{\rm P}^2 f'(\eta) q, \qquad \frac{\Lambda_\infty}{M_N^2} = \frac{f'(\eta)}{2\eta} q$$

on a maximally symmetric vacuum branch. Requiring the coefficient relating Λ_∞ / M_N^2 to q to be independent of the arbitrary branch coordinate η implies $f'(\eta) = 2c\eta$, hence

$$f(\eta) = c\eta^2 + f_0.$$

The minimal one-source action sets the additional η-independent source term f_0 to zero. Branch-unit covariance therefore fixes the η^2 form, while the overall coefficient c remains the compact-to-global pairing; Eq. (37) takes the proposed unit choice $c = 1$.

The variations are

$$\delta\Lambda_b : \quad F_4 = \star 1, \tag{50}$$

$$\delta A_3 : \quad d\left[\Lambda_b - \frac{\eta^2 M_{\rm P}^4}{8\pi} q \right] = 0, \tag{51}$$

$$\delta\eta : \quad \langle R \rangle = 4\eta M_{\rm P}^2 q, \tag{52}$$

$$\delta g^{\mu\nu} : \quad \frac{\eta M_{\rm P}^2}{8\pi} G_{\mu\nu} + \Lambda_b g_{\mu\nu} = T_{\mu\nu}. \tag{53}$$

Taking the averaged trace of the metric equation and eliminating Λ_b gives Eq. (39). The normalization is therefore fixed at the action level within this proposed completion; the statement $\Lambda_\infty / M_N^2 = q$ is a theorem of this action on a maximally symmetric vacuum branch, while $M_N = M_{\rm P}$ is the numerical branch convention used in the main text. The result is specific to the action in Eq. (37) and its stated branch parametrization.

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